Netherlands Institute for Radio Astronomy

Introduction to Radio Astronomy

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ASTRON is part of the Netherlands Organisation for Scientific Research (NWO)

Preamble

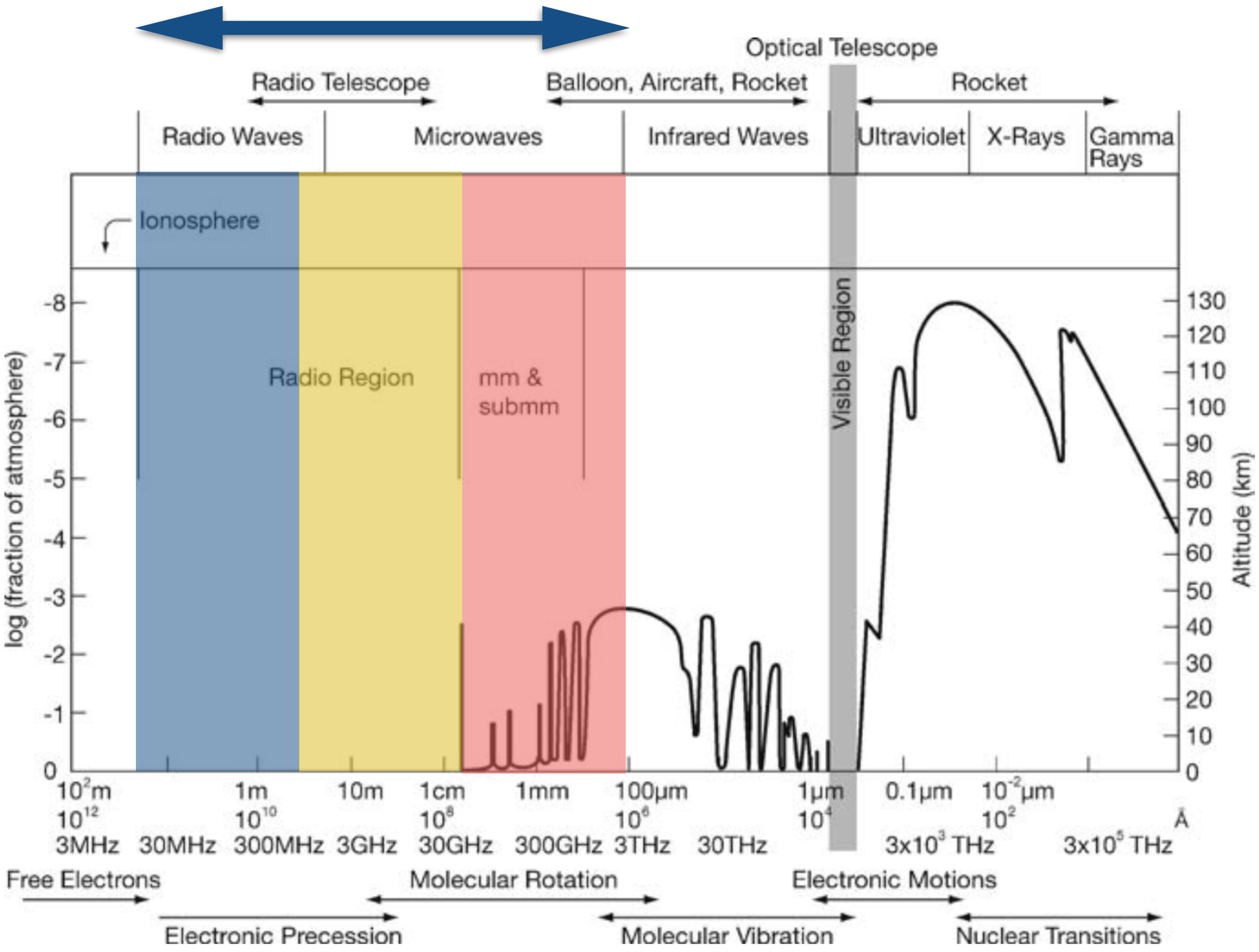
- **AIM:** This lecture aims to give a general introduction to radio astronomy, focusing on the issues that you must consider for single element telescopes that make up an interferometer

- **OUTLINE:**

  1. The radio sky and historical developments
  2. The response of a dipole antenna
  3. The response of a dish antenna

1.1 The Radio Window

- Radio Astronomy is the study of radiation from celestial sources at frequencies between $\nu \sim$ 10 MHz to 1 THz (10^7 Hz to 10^{12} Hz).



- The observing window is constrained by atmospheric absorption / emission and refraction.
  1) Charged particles in the ionosphere reflects radio waves back into space at < 10 MHz.
  2) Vibrational transitions of molecules have similar energy to infra-red photons and absorb the radiation at > 1 GHz (completely by ~300 GHz).

1.2 The low-frequency cut-off

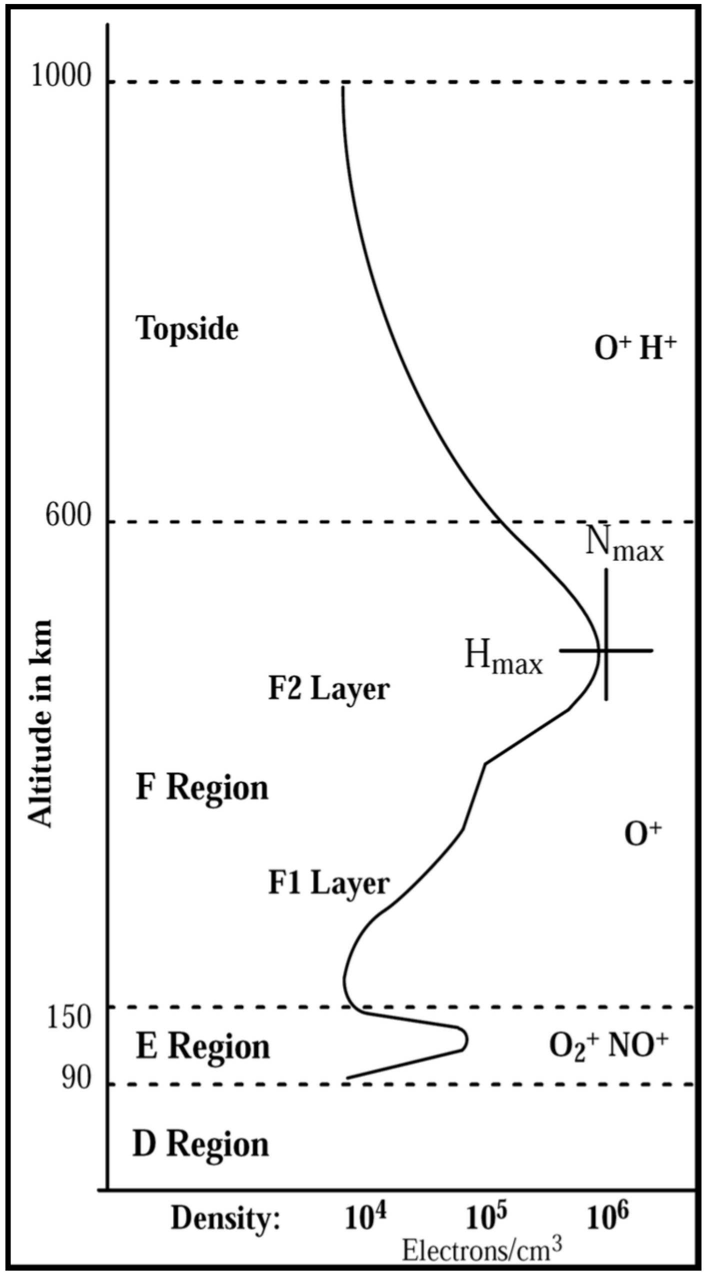
- The ionosphere consists of a plasma of charged particles (conducting layers), that has an effective refractive index of,

$$n^2 = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2} = 1 - \left(\frac{\lambda}{\lambda_\mathrm{p}}\right)^2$$

where, the plasma frequency is defined as,

$$\nu_\mathrm{p}[\mathrm{Hz}] = \frac{\omega_\mathrm{p}}{2\pi} = \left(\frac{N_\mathrm{e} e^2}{4\pi^2 \epsilon_0 m}\right)^{1/2} = 8.97 \times 10^3 \sqrt{\frac{N_e}{[\mathrm{cm}^{-3}]}}$$

when $\omega < \omega_\mathrm{p}$, there is no propagation, i.e. total reflection.



Worked example: What is the cut-off frequency for LOFAR observations carried out when the electron density is $N_e = 2.5 \times 10^5$ cm^{-3} (night time) and $N_e = 1.5 \times 10^6$ cm^{-3} (day time)?

$$\nu_{\rm p}[{\rm Hz}] = 8.97 \times 10^3 \sqrt{\frac{2.5 \times 10^5}{[{\rm cm}^{-3}]}} = 4.5 \ {\rm MHz} \qquad \text{(night time)}$$

$$\nu_{\rm p}[{\rm Hz}] = 8.97 \times 10^3 \sqrt{\frac{1.5 \times 10^6}{[{\rm cm}^{-3}]}} = 11 \ {\rm MHz} \qquad \text{(day time)}$$

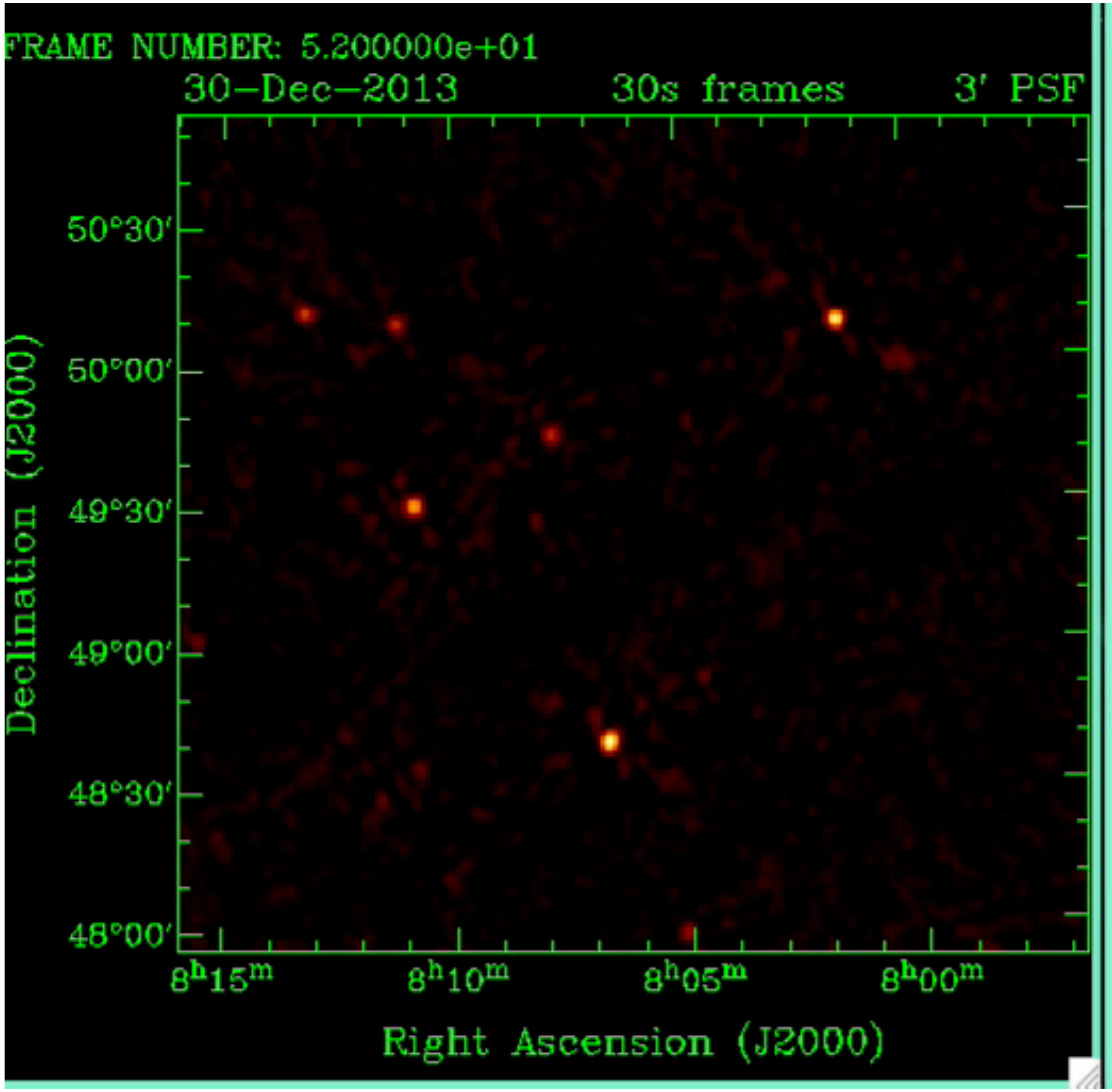
- At frequencies,
    1. $\omega < \omega_{\rm p}$: n^2 is negative, reflection ($\nu < 10$ MHz),
    2. $\omega > \omega_{\rm p}$: n^2 is positive, refraction (10 MHz $< \nu <$ 10 GHz),
    3. $\omega \gg \omega_{\rm p}$: n^2 is unity ($\nu >$ 10 GHz).

- The observing conditions are dependent on the electron density, i.e. the solar conditions (space weather), since the ionisation is due to the ultra-violet radiation field from the Sun,

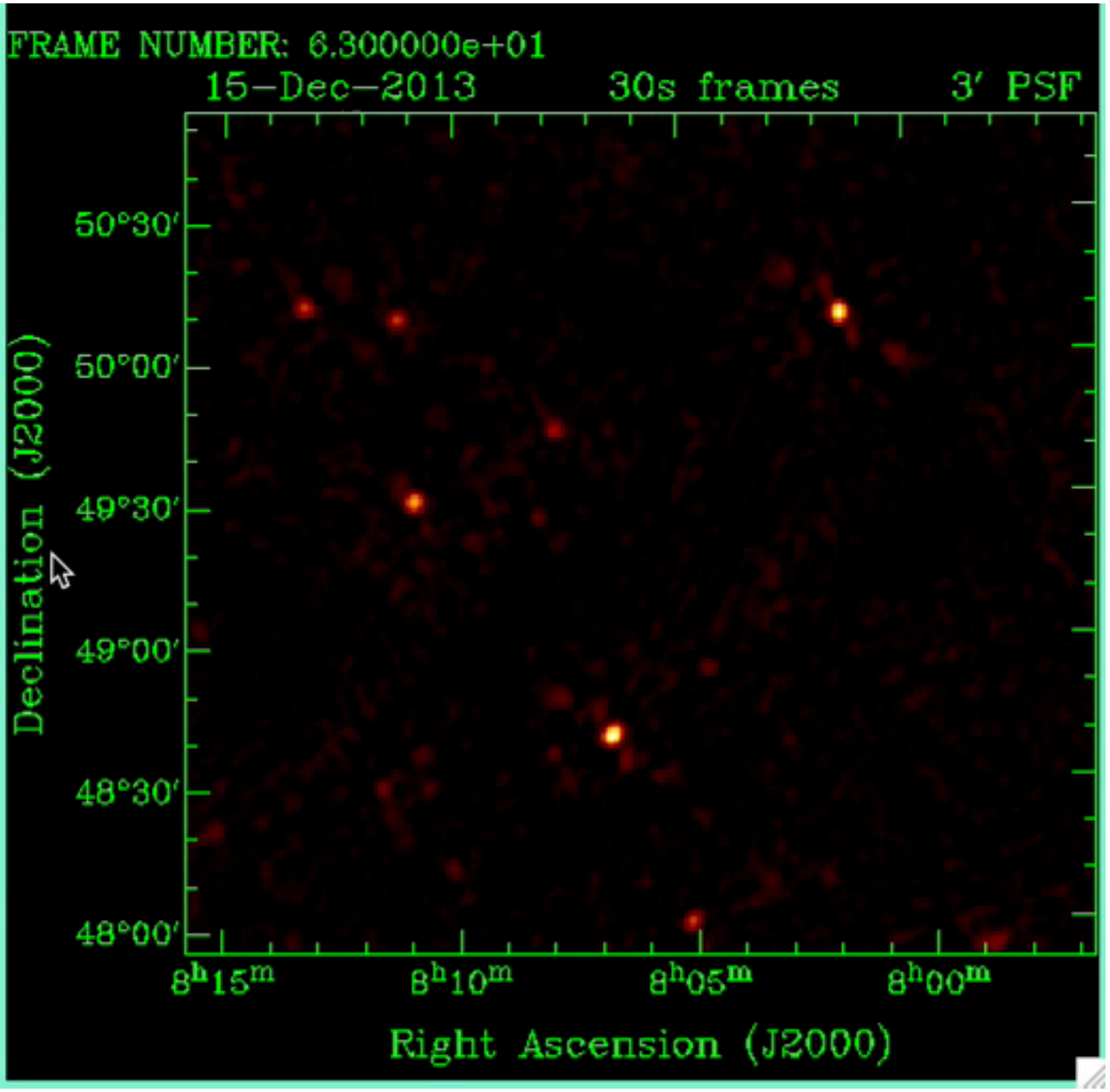
$$O_2 + h\nu \rightarrow O_2^{+*} + e^-$$

$$O_2 + h\nu \rightarrow O^+ + O + e^-$$

- Bad observing conditions



- Good observing conditions



1.3 The high-frequency cut-off (absorption)

- Molecules in the atmosphere can absorb the incoming radiation, but also emit radiation (via thermal emission).

- Mass absorption co-efficient *(k)*: From atomic and molecular physics, define for various species, *i*,

Cross-section (cm^2)

Number density of particles (cm^{-3})

Mass attenuation co-efficient (cm^2 g^{-1})

$$k_i = \frac{\sigma\, n_i}{r_i\, \rho_0}$$

Mixing ratio (= ρ_i/ρ_0)

Mass density of air (g cm^{-3})

- Optical depth *(τ)*: A measure of the absorption / scattering (attenuation) of electromagnetic radiation in a medium (probability of an interaction),

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} n_i(z)\, \sigma\, dz = \int_{z_0}^{\infty} r_i(z)\, \rho_0(z)\, k_i(\lambda)\, dz$$

or, in terms of the linear absorption co-efficient *(κ)*,

linear absorption co-efficient (cm^{-1})

Mass density of species i (g cm^{-3})

$$\tau_i(\lambda, z_0) = \int_{z_0}^{\infty} \kappa(\lambda, z)\, dz \qquad \text{where} \quad \kappa(\lambda, z) = k_i(\lambda)\, \rho_i(z)$$

Mass attenuation co-efficient (cm^2 g^{-1})

- The attenuation of an incident ray of intensity I_0, received at altitude z_0, summed over all absorbing species is,

$$I(z_0) = I_0 \exp\left[-\sum_i \tau(\lambda, z_0)\right] = I_0 \exp\left[-\tau(z)\right]$$

Where, for convenience, we consider all species together and define the optical depth as a function of zenith angle, $\tau(z)$.

Worked example: What is the optical depth for sky transparencies of 0.5, 0.1 and 0.01?

Rearrange, in terms of τ, and evaluate, $\tau = -\ln\left(\frac{I(z_0)}{I_0}\right)$

$$\tau_{0.5} = -\ln(0.5) = 0.69$$

$$\tau_{0.1} = -\ln(0.1) = 2.3$$

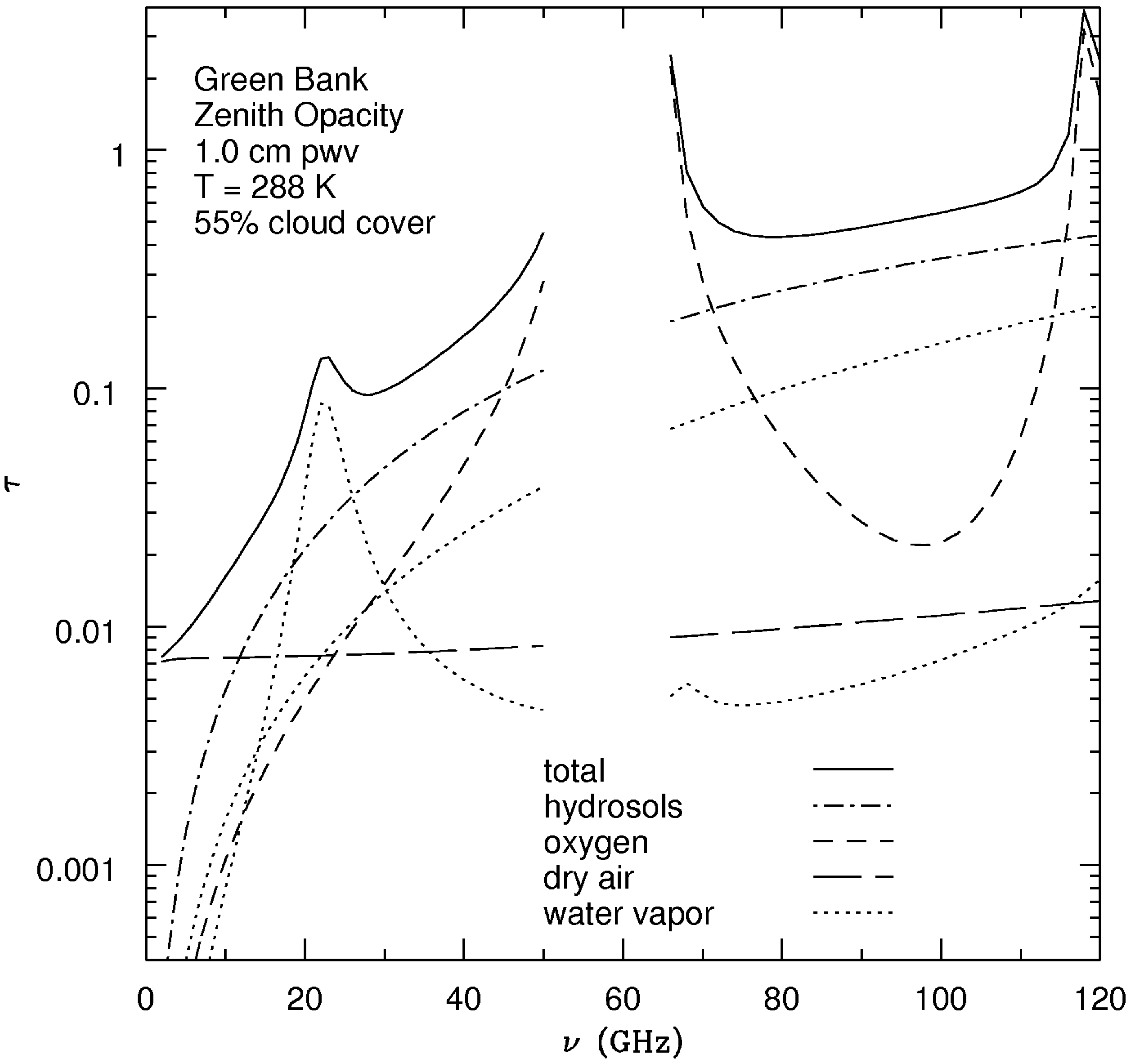
$$\tau_{0.01} = -\ln(0.01) = 4.6$$

- Note that the opacity changes with the path length, and so depends on the airmass $X(z)$, which assuming a plane parallel atmosphere,

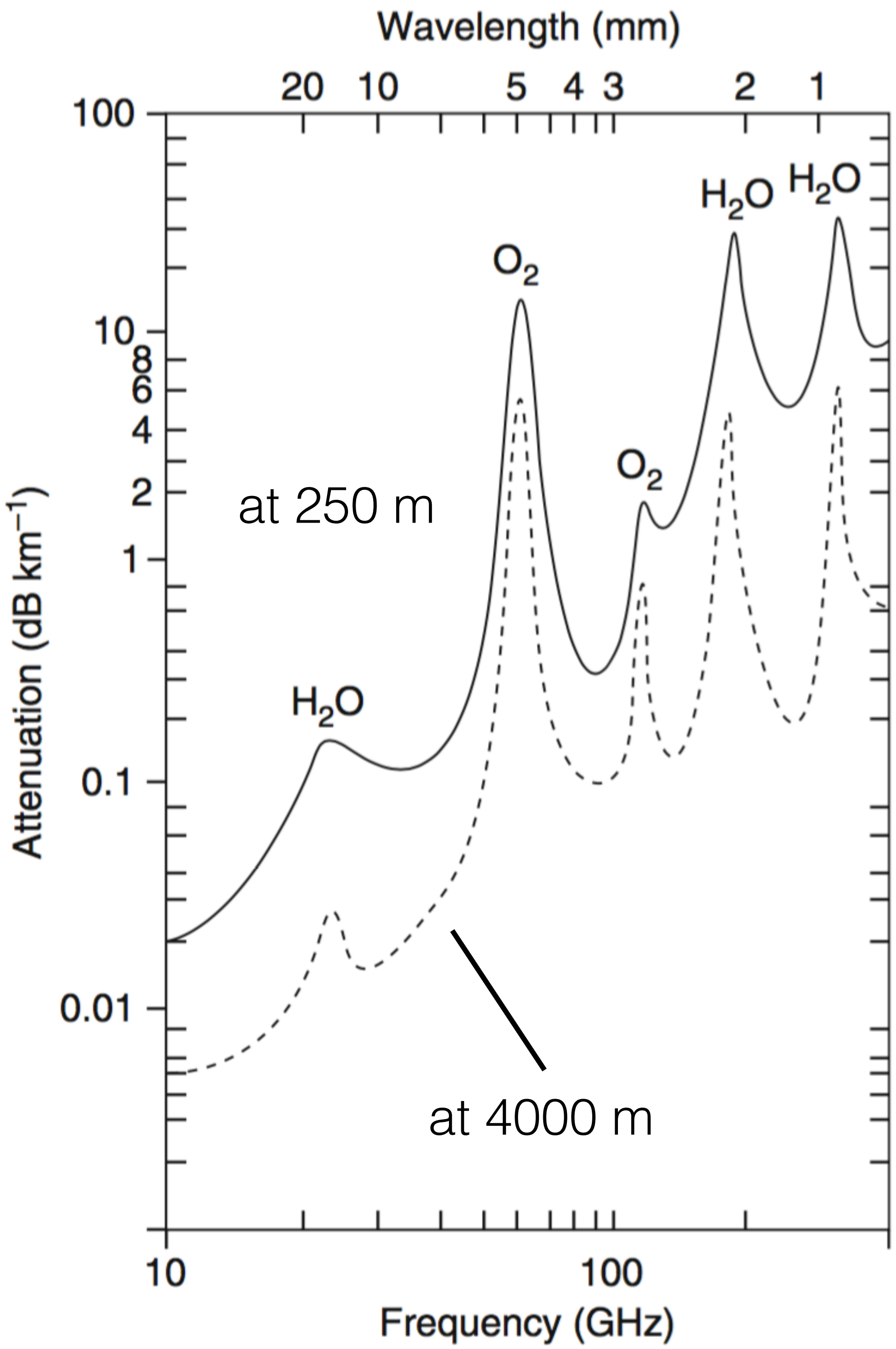
$$\tau(z) = \underbrace{\tau_0}_{\text{Optical depth at Zenith}} \cdot \underbrace{X(z)}_{\text{Airmass}} \quad \text{where} \quad X(z) = \sec(\underbrace{z}_{\text{Zenith angle}})$$

- The atmosphere is not completely transparent at radio wavelengths, but $\tau(z)$ varies with frequency ν.

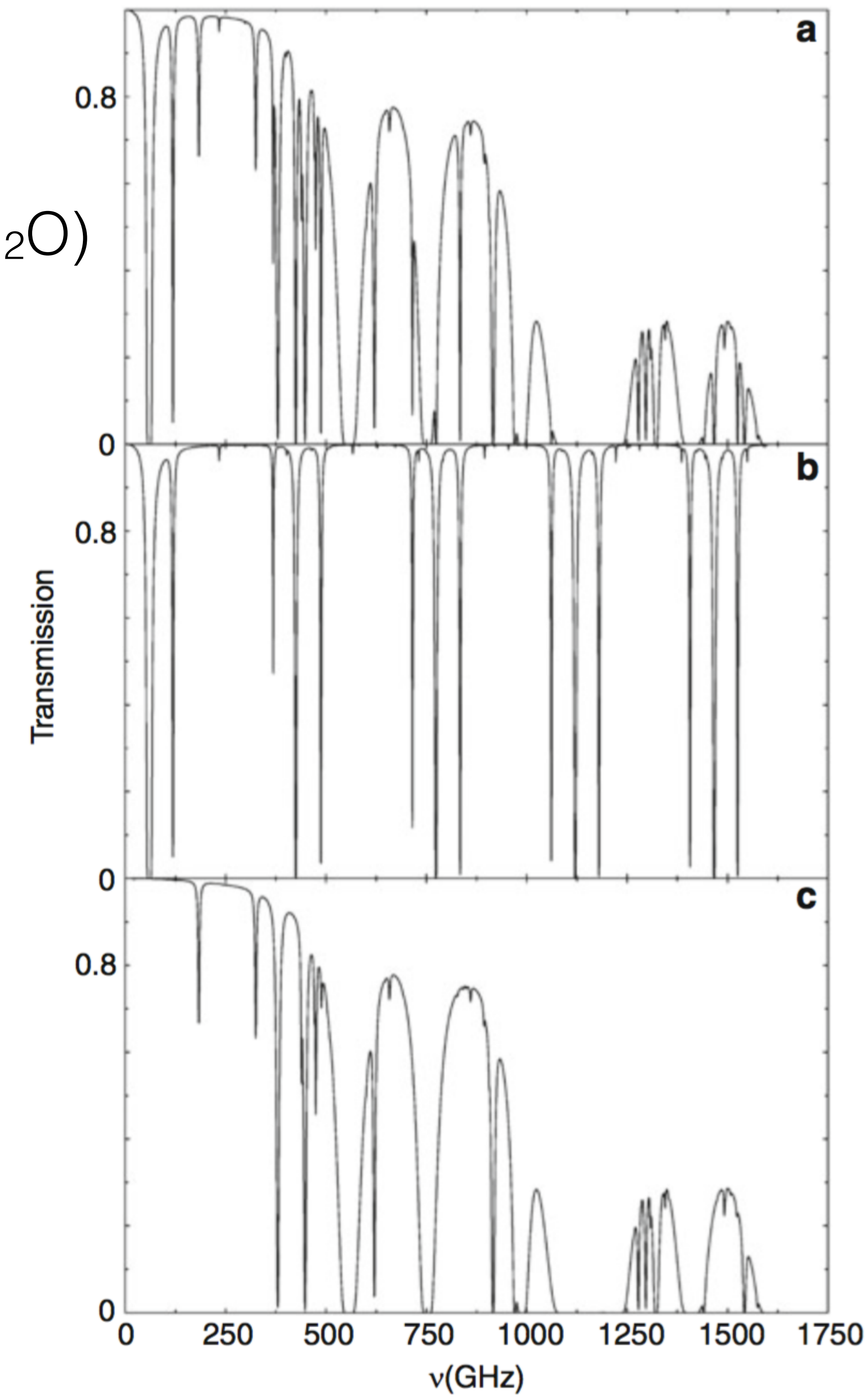
- Zenith opacity is the sum of several component opacities at cm λ.
  - Broadband (continuum) opacity: dry air. $\tau_z \approx 0.01$ and almost independent of ν.
  - Molecular absorption: O_2 has rotational transitions that absorb radio waves and are opaque ($\tau_z \gg 1$) at 52 to 60 GHz.
  - Hydrosols: Water droplets (radius ≤ 0.1 mm) suspended in clouds absorb radiation (proportional to λ^{-2}).
  - Water vapour: Emission line at $\nu \approx 22.235$ GHz is pressure broadened to $\Delta\nu \sim 4$ GHz width + "continuum" absorption from the "line-wings" of very strong H_2O emission at infrared wavelengths (proportional to λ^{-2}).



- The zenith optical depth is dependent on the path length through the material.
  - Higher altitude: Move above the water vapour layer (> 4 km).
  - Drier locations: Move to regions with low water vapour.



(note τ = 2.3 x Attenuation)



total ($O_2 + H_2O$)

O_2

H_2O

1.3 The high-frequency cut-off (emission)

- A partially absorbing atmosphere also emits radio noise that can de-grade ground based observations. We can define the total system noise power as an *equivalent noise temperature*,

$$P = \frac{E}{\Delta t} = k\,T\,\Delta\nu$$

in terms of *spectral power*,

Spectral power (W Hz^{-1})

System temperature (Receivers; Sky, Ground; etc)

$$P_\nu = k\,T_{\rm sys}$$

Boltzmann constant = 1.38 x 10^{-23} m^2 kg s^{-2} K^{-1}

where,

Noise from Radio background (Galaxy, CMB, etc)

Noise from ground emission

Noise from injected noise

Noise from the receiver (Dominates)

$$T_{\rm sys} = T_{\rm bg} + T_{\rm sky} + T_{\rm spill} + T_{\rm loss} + T_{\rm cal} + T_{\rm rx}$$

Noise from atmospheric emission

Noise from losses at receiver

- The contribution from the sky opacity to the sky temperature is,

Atmospheric kinetic temperature (≡ 300 K)

Emitted sky temperature (K)

$$T_{\rm sky} = T_{\rm atm}\left[1 - \exp(-\tau_\nu)\right]$$

Optical depth

- Don't want $T_{\rm sky}$ to dominate our noise budget, need to minimise $T_{\rm atm}$ and τ_ν by observing in cold and dry locations (winter; high alt), especially at high frequencies.

Worked example: Using the total opacity data for the Green Bank Telescope (West Virginia; USA; 2800 m) and $T_{\rm atm}$ = 288 K, what is $T_{\rm sky}$ at ν = 5 GHz, 22 GHz and 115 GHz?

How does this compare with the typical receiver temperature, $T_{\rm rx}$ ~ 30 K?

- At ν = 5 GHz, $\tau_Z \sim 0.007$, $T_{\rm sky} = 288\left[1 - \exp(-0.007)\right] \sim 2$ K (Good)
- At ν = 22 GHz, $\tau_Z \sim 0.15$, $T_{\rm sky} = 288\left[1 - \exp(-0.15)\right] \sim 40$ K (Bad)
- At ν = 115 GHz, $\tau_Z \sim 0.8$, $T_{\rm sky} = 288\left[1 - \exp(-0.8)\right] \sim 160$ K (Bad)

Key concept: The partially transparent atmosphere allows radio waves to be detected from ground-based telescopes, but also attenuates the signal due to absorption / scattering, and also adds noise to the measured signal.

1.4 Early Radio Astronomy

- The first detection of radiation at radio wavelengths was not made until 1932 due to,
  i) limitations of technology (our eyes), but then the communication era started,
  ii) the expectation that celestial objects would be too faint.

- The spectral brightness B_ν at frequency ν of a blackbody object (stars) is given by Planck's law.

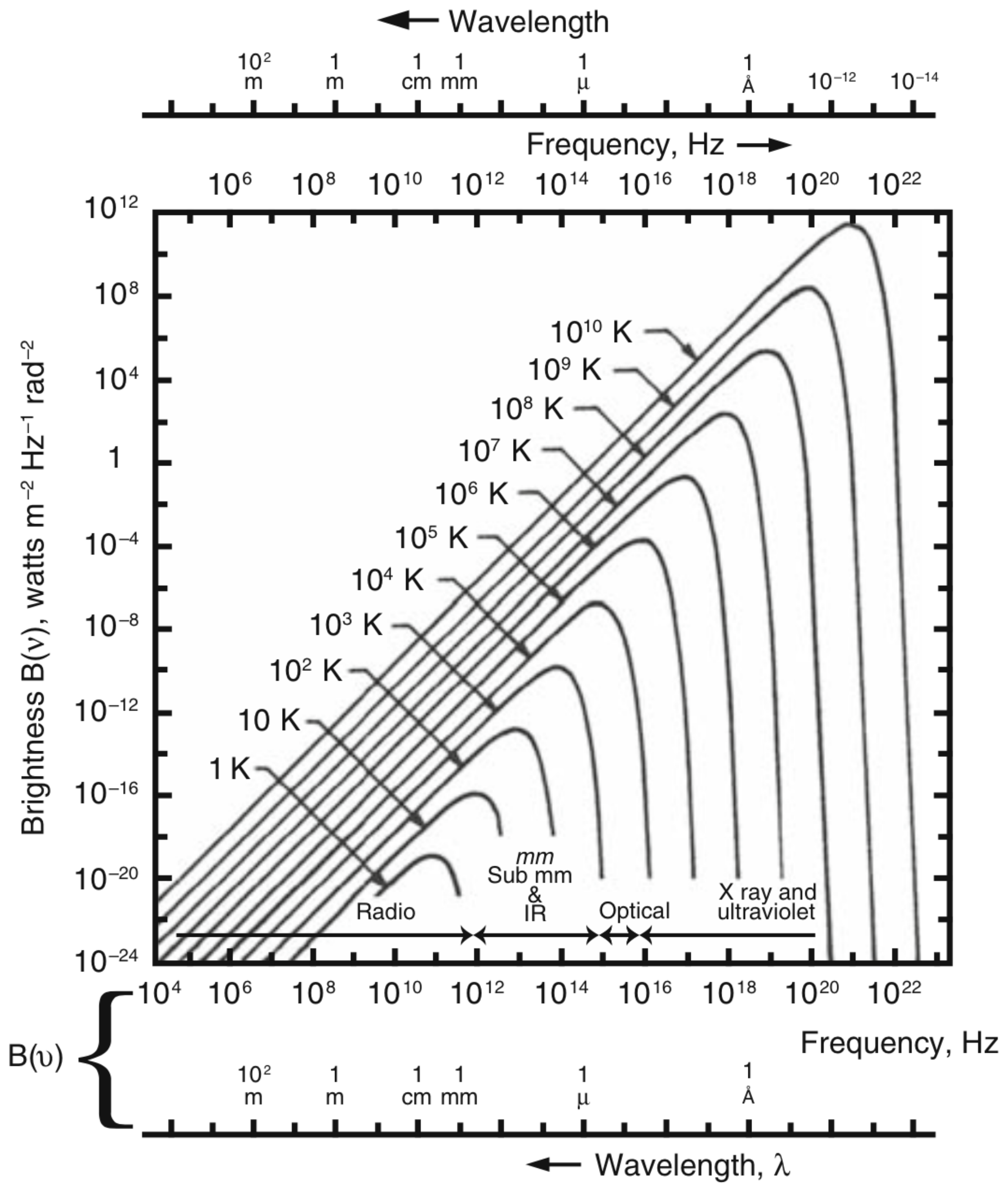
Spectral brightness (W m^{-2} Hz^{-1} sr^{-1})

Planck constant = 6.626 x 10^{-34} m^2 kg s^{-1}

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp \frac{h\nu}{kT} - 1}$$

Speed of light constant = 3 x 10^8 m s^{-1}

Absolute temperature (K)



- In the low frequency radio limit, $h\nu / kT \ll 1$.

Worked example: Does the low-frequency limit work for the photosphere of the Sun, which has T = 5800 K? At ν = 1 GHz,

$$\frac{h\nu}{kT} = \frac{6.626 \times 10^{-34} * 1 \times 10^{9}}{1.38 \times 10^{-23} * 5800} = 8 \times 10^{-6}$$

- Using this property, we can replace the exponential term using the Taylor expansion,

$$\exp\left(\frac{h\nu}{kT}\right) - 1 \approx 1 + \frac{h\nu}{kT} + ... - 1 \qquad \left(e^{x} \approx 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right)$$

to give the Rayleigh-Jeans approximation to the Planck function at low-frequencies,

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\frac{kT}{h\nu} = \frac{2kT\nu^2}{c^2} = \frac{2kT}{\lambda^2}$$

- Flux-density (S_ν): The power received per unit detector area in a unit bandwidth ($\Delta\nu$ = 1 Hz) at frequency ν. The units are W m^{-2} Hz^{-1}.

- The flux-density received from a celestial source of brightness $B_\nu(T)$ and subtending a very small angle $\Omega \ll 1$ sr, is approximately,

$$S_\nu = B_\nu \Omega$$

Worked example: What is the flux-density at *ν* = 1 GHz of a black body with temperature *T* = 5800 K and size *R* ≈ 7 x 10^{10} cm (the Sun) at about 1 parsec (d ≈ 3 x 10^{18} cm).

$$B_\nu(T) \approx \frac{2kT\nu^2}{c^2} = \frac{2 * 1.38 \times 10^{-23} * 5800 * (1 \times 10^9)^2}{(3 \times 10^8)^2}$$

$$= 1.78 \times 10^{-18} \text{ Wm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$$

The spectral brightness is an intrinsic property of the source (independent of distance).

The solid angle subtended by the source is dependent on the distance,

$$\Omega = \frac{\pi R^2}{d^2} \approx \frac{\pi * (7 \times 10^8)^2}{(3 \times 10^{16})^2} \approx 1.71 \times 10^{-15} \text{ sr}$$

The flux-density is therefore,

$$S_\nu = B_\nu \Omega \approx 3 \times 10^{-33} \text{ W m}^{-2} \text{ Hz}^{-1}$$

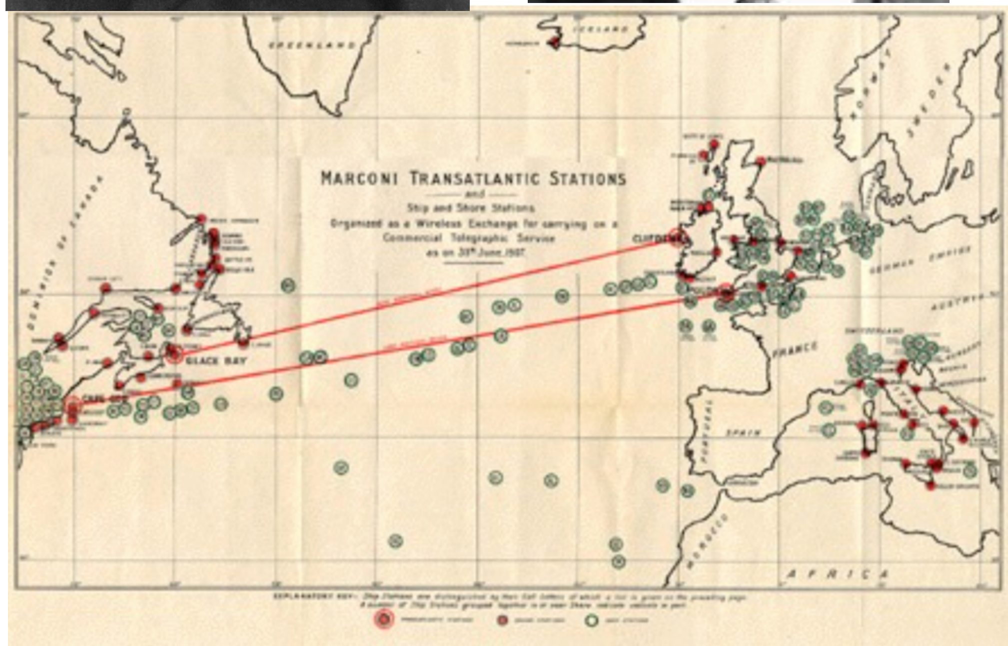
- This flux density is too small for even todays telescopes to detect (easily), so the thermal emission from stars was thought to be impossible to detect at radio wavelengths, but…

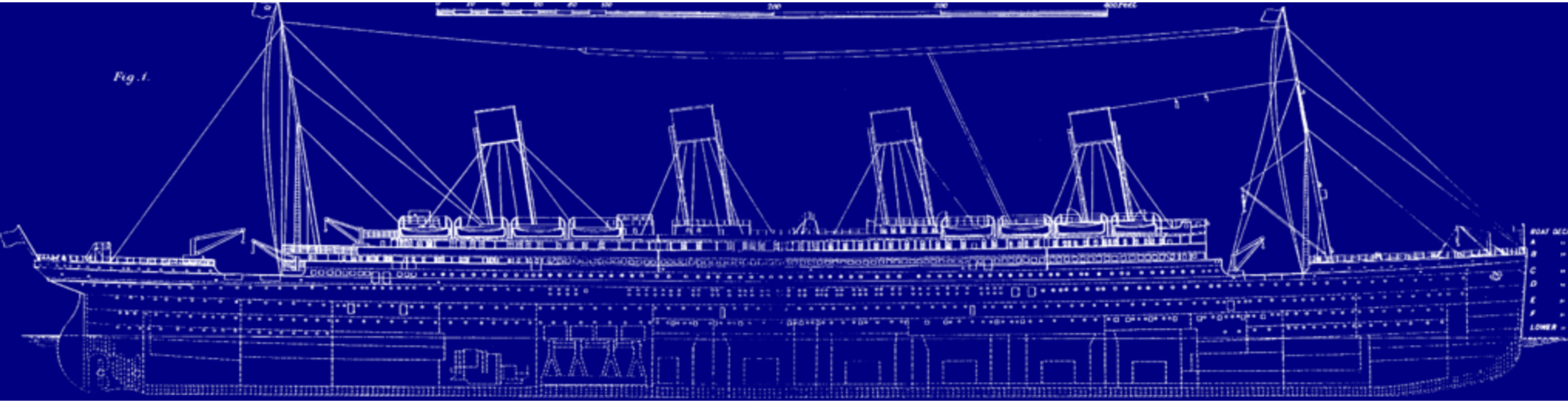
- Long distance communication developed by Marconi & Ferdinand Braun - Nobel Prize 1909

Evolution of frequency over the years

- pre-1920: <100 kHz.
- ca. 1920: shift to 1.5 MHz.
- post-1920: 10s of MHz (more voice channels, less effected by the ionosphere and thunderstorms).

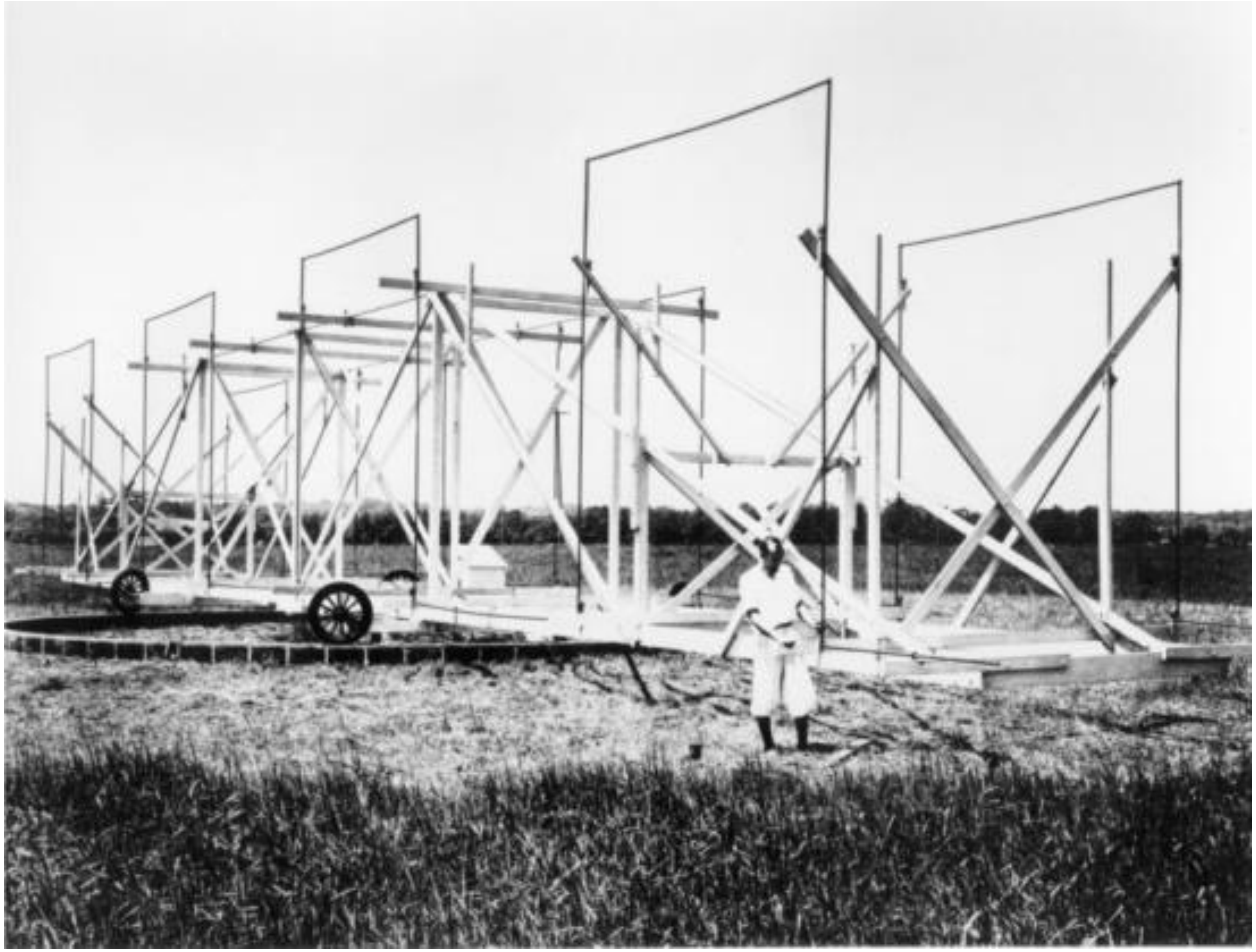
- Research labs sprung up in early-1900s

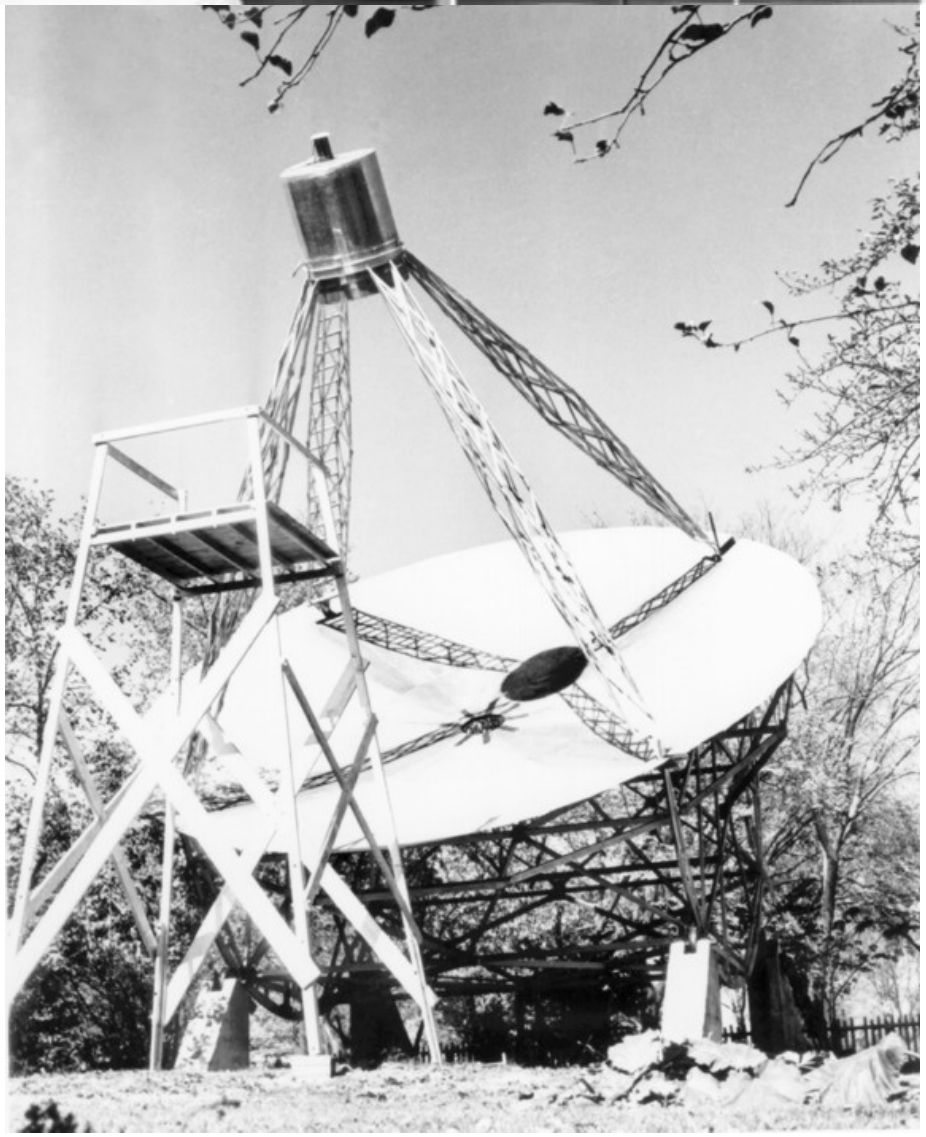




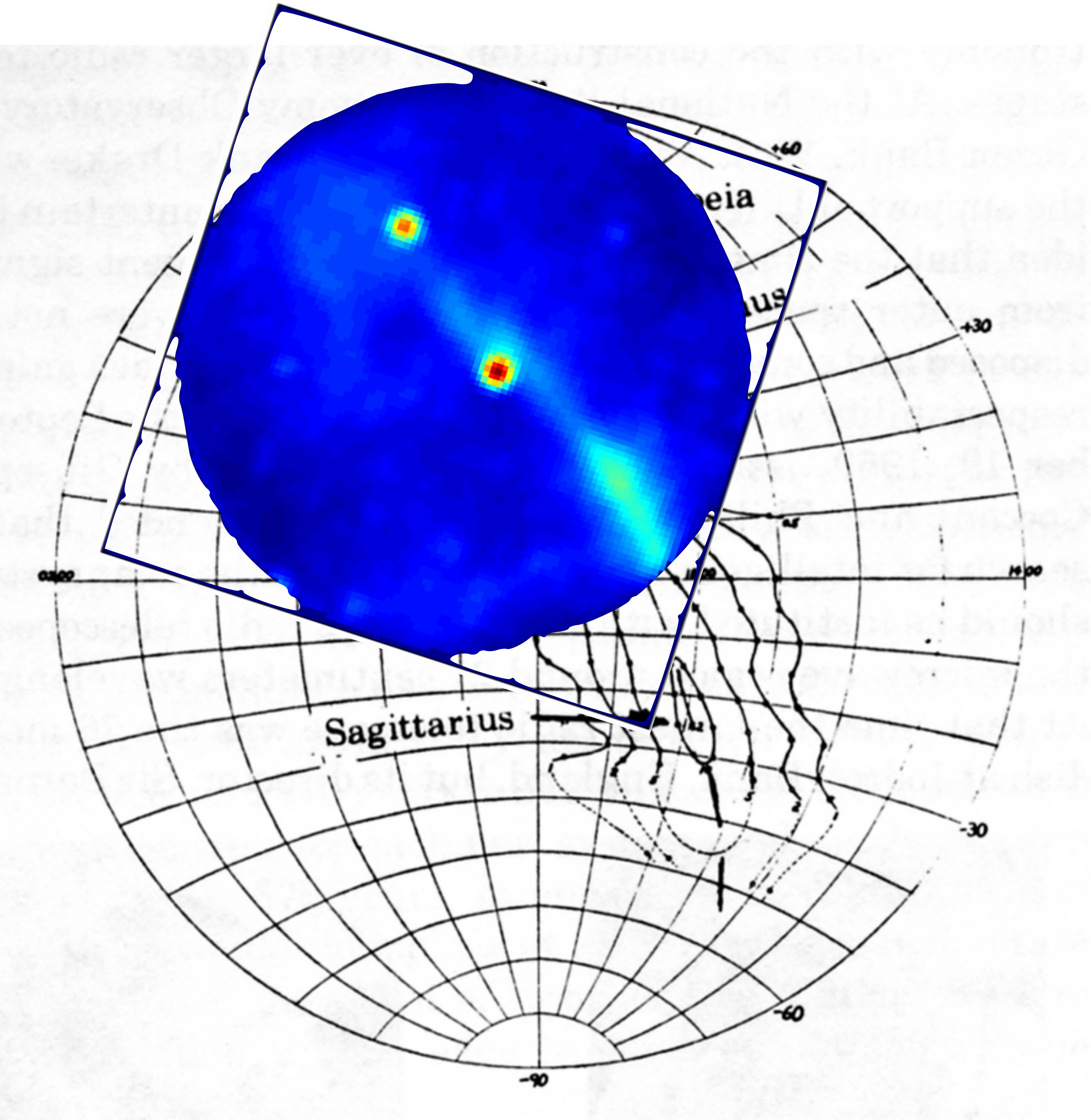
- Karl Jansky (1933, published) discovered a radio signal at 20.5 MHz that varied steady every 23 hours and 56 minutes (Sidereal day).

“The data give for the co-ordinates of the region from which the disturbance comes, a right ascension of 18 hours and declination -10 degrees.” He had detected the Galactic Centre.





- Grote Reber (1937-39), using his own 10 m telescope, made no detection at 3300 and 910 MHz, ruling out a Planck spectrum (B_ν propto ν^2).
- Detection made at 150 MHz, confirming Jansky's result and finding the spectrum must be non-thermal.



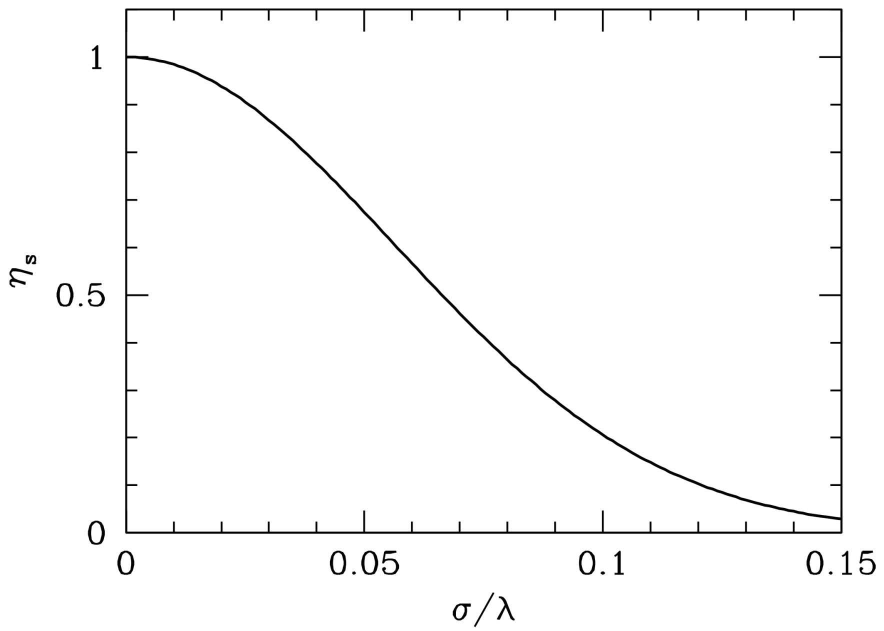
Key concept: Radio emission from celestial objects can be measured and it can be both thermal and non-thermal in origin.

1.4 Radio telescopes and interferometers

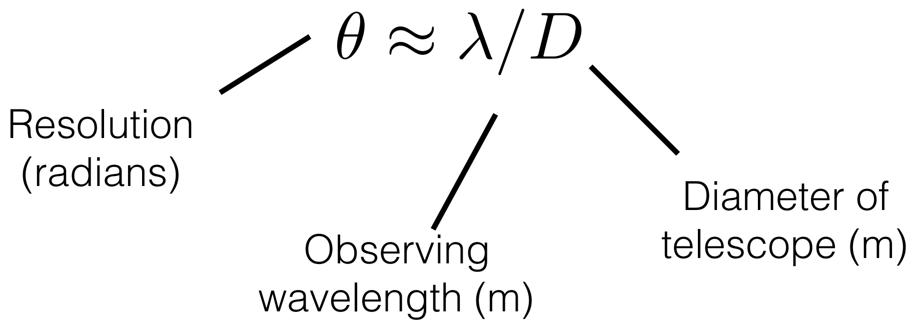
- Radio telescopes are designed in a different way to optical telescopes, and the radio range is so broad (5 decades in frequency) that different telescope technologies can be used.

- The surface accuracy of a reflector is proportional to λ / 16
    - cm (1 GHz) -> surface accuracy of ~ 2 cm
    - mm (100 GHz) -> surface accuracy of ~ 200 μm.
    - Optical (0.55 μm) -> surface accuracy of ~ 0.034 μm.

$$\boxed{\frac{P(\delta)}{P_0} = \eta_s = \exp\left[-\left(\frac{4\pi\sigma}{\lambda}\right)^2\right]}$$



- Large single-element radio telescopes can be constructed cheaply, but have limited spatial resolution,



$$\theta \approx \lambda/D$$

Resolution (radians)

Observing wavelength (m)

Diameter of telescope (m)

Worked example: What is the spatial resolution (in arcseconds) of the $D = 300$ m Arecibo telescope, operating at $\nu = 5$ GHz?

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \text{ m}}{5 \times 10^9 \text{ Hz}} = 0.06 \text{ m}$$

$$\theta \sim \frac{0.06 \text{ m}}{300 \text{ m}} = 0.0002 \text{ radians} \equiv 41 \text{ arcsec}$$

Arecibo, Puerto Rico: 300 m

Green Bank Telescope: 110 m

- Interferometric techniques have been developed to combine several single-element telescopes into a multi-element array. Now, the resolution is limited by the distance between the elements

$$\theta \approx \lambda / D$$

Resolution (radians)

Observing wavelength (m)

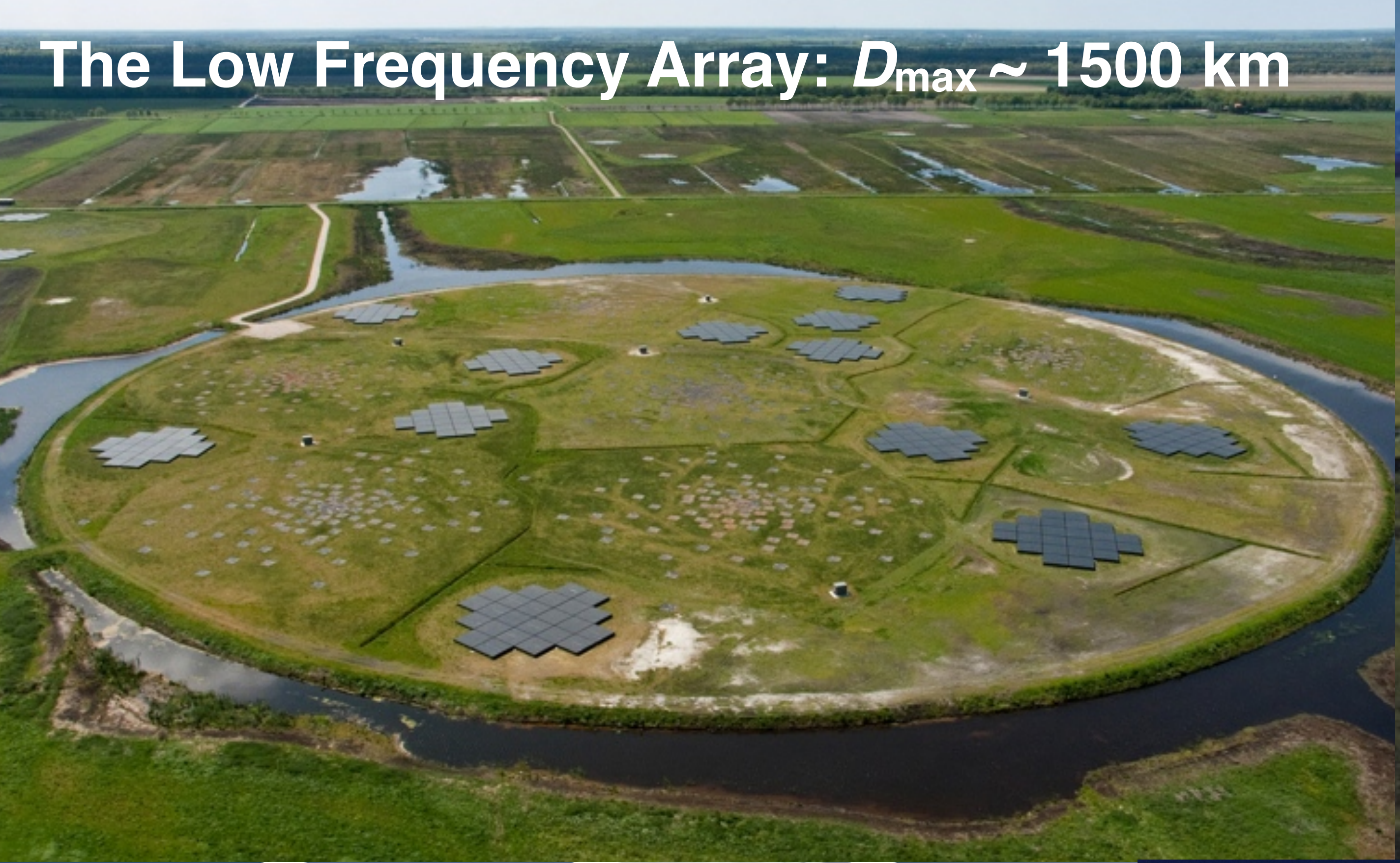
Distance between telescopes (m)

Worked example: What is the spatial resolution (in arcseconds) of the Very Long Baseline Array operating at *v* = 5 GHz? The longest distance between telescopes is D_{max} = 8611 km.

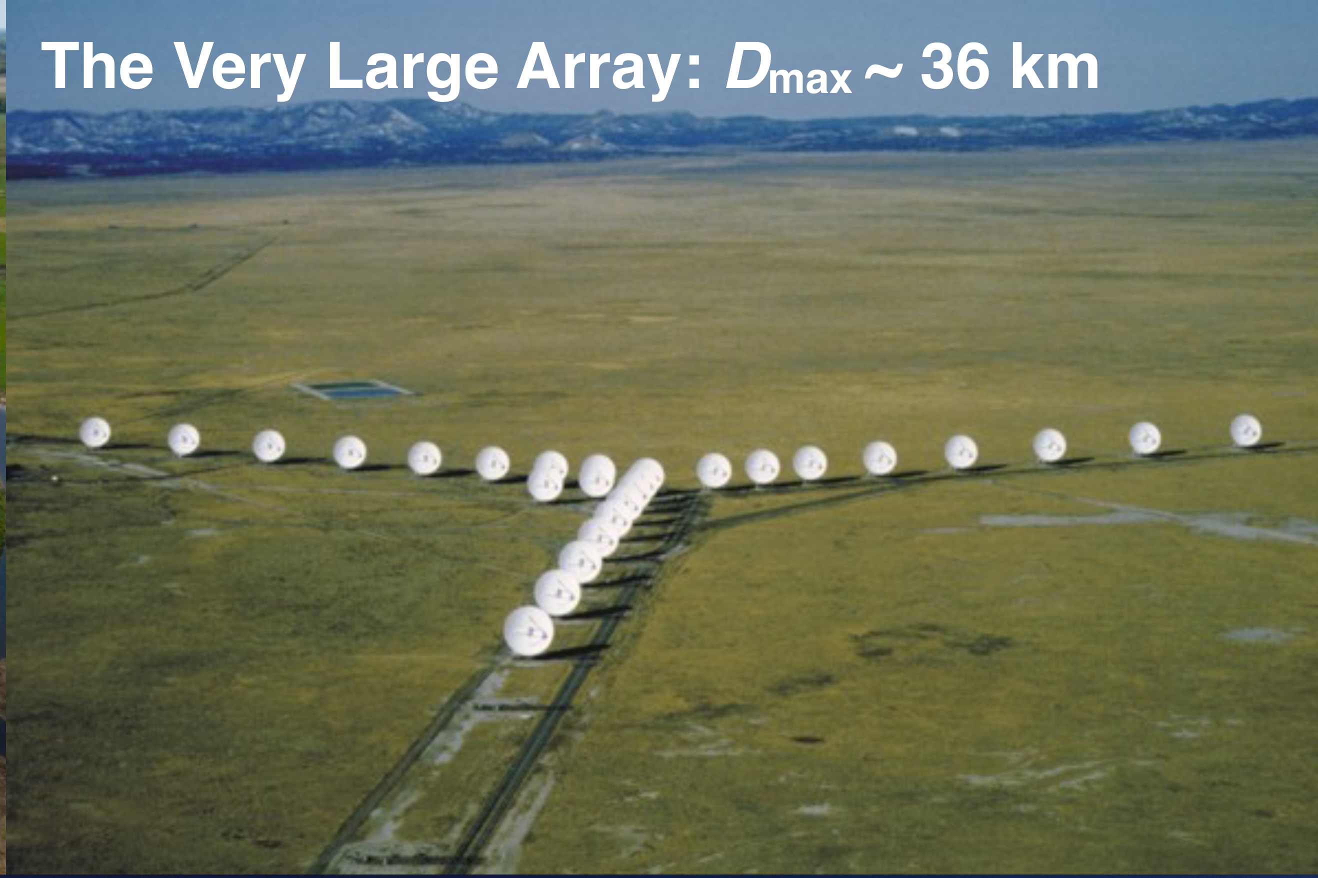
$$\theta \sim \frac{0.06 \text{ m}}{8.611 \times 10^6 \text{ m}} = 7 \times 10^{-9} \text{ rad}$$

$$\frac{180}{\pi} * 3600 * 7 \times 10^{-9} = 0.00144 \text{ arcsec}$$

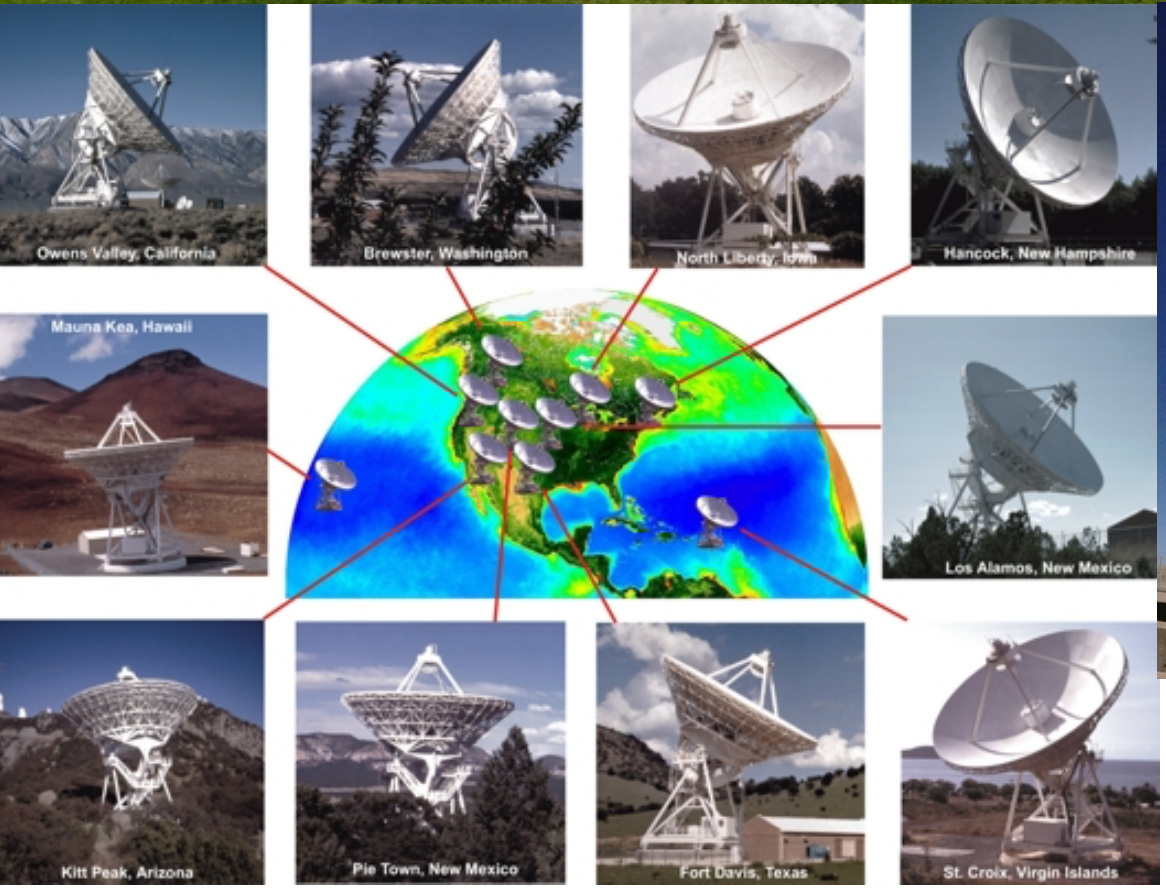
Key Concept: Radio interferometry can provide the highest angular resolution imaging possible in astronomy.



The Low Frequency Array: $D_{\max} \sim 1500$ km



The Very Large Array: $D_{\max} \sim 36$ km



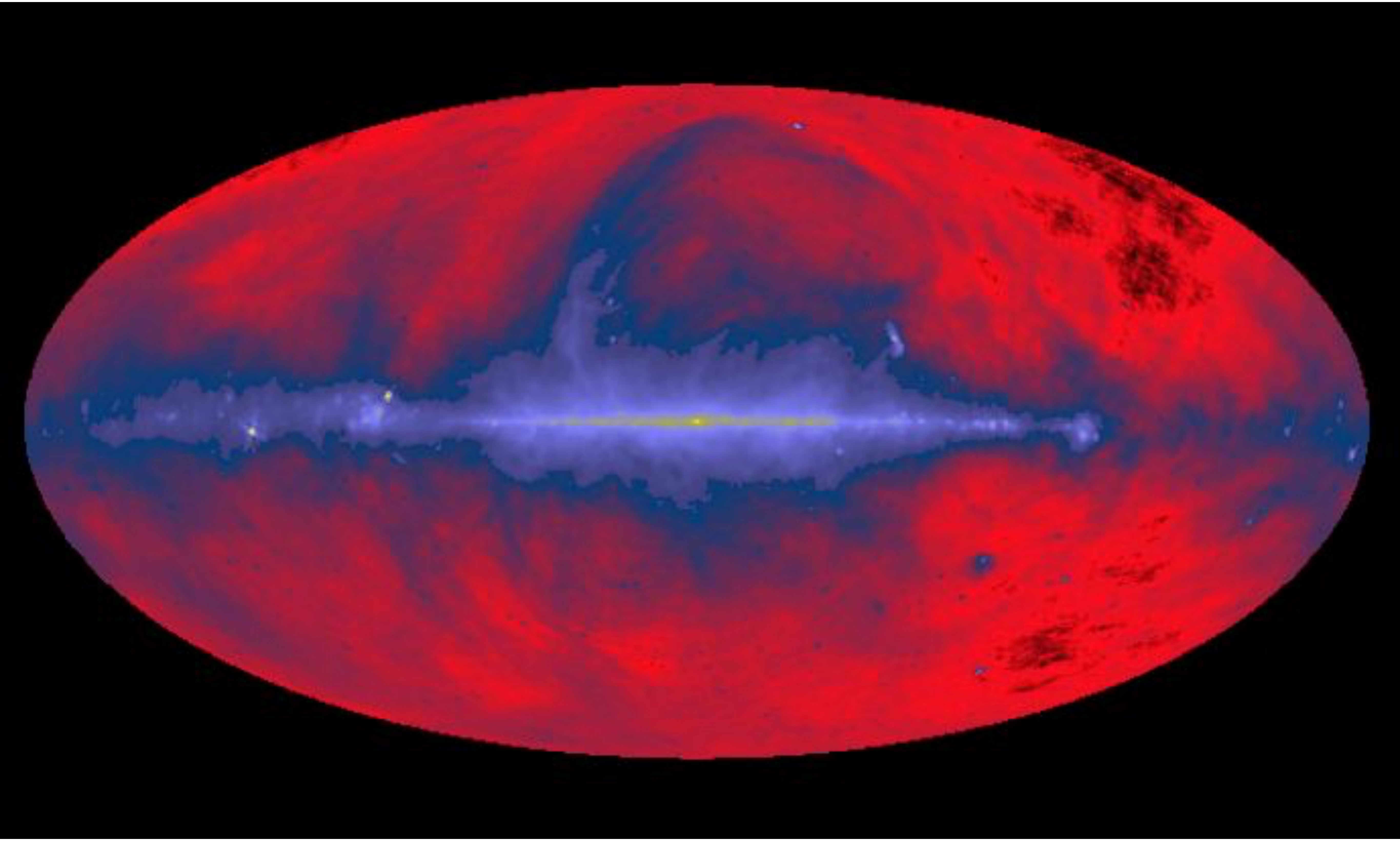
Atacama Large Millimetre Array: $D_{\max} \sim 16$ km

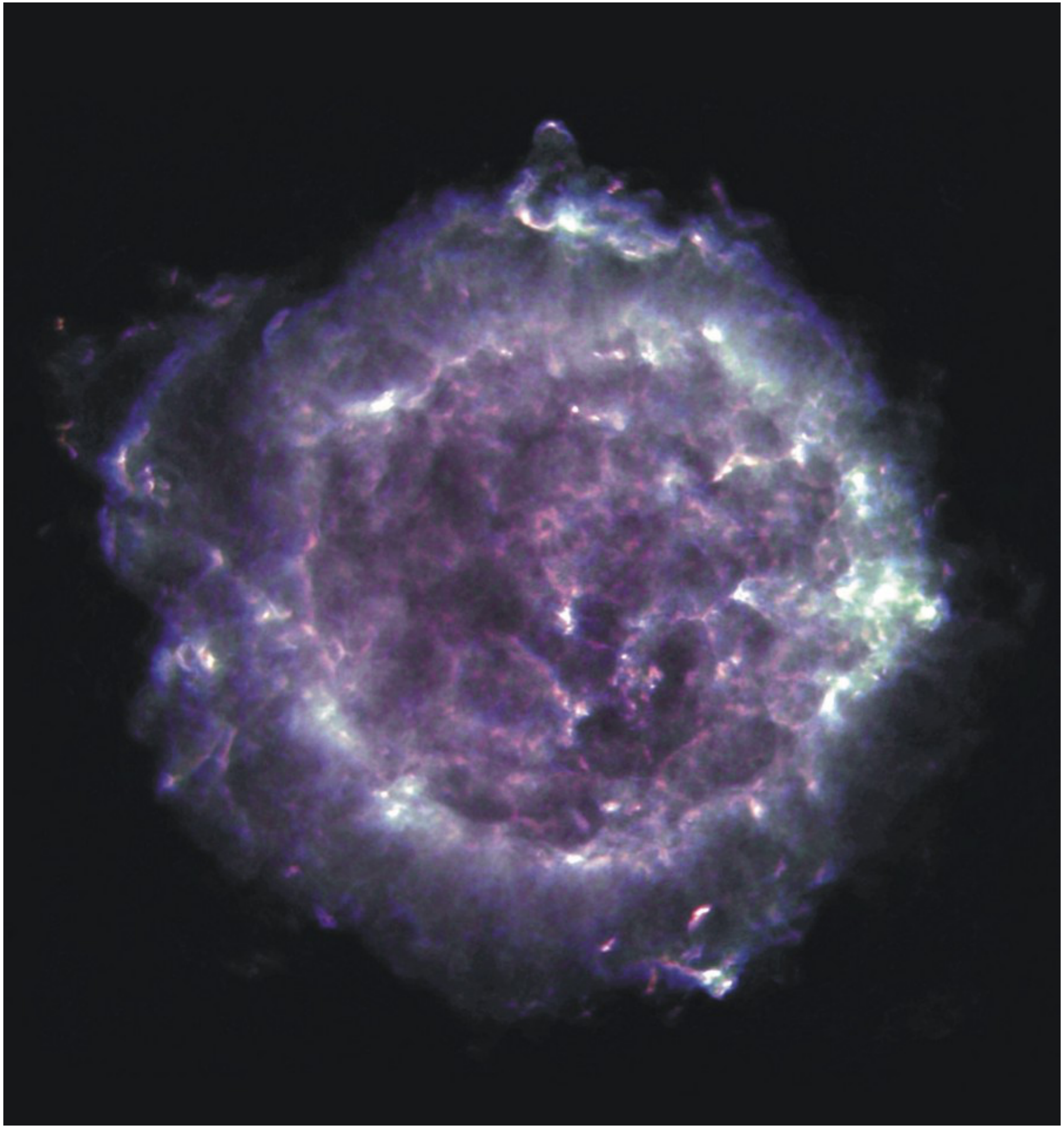
The Very Long Baseline Array: $D_{\max} \sim 9000$ km

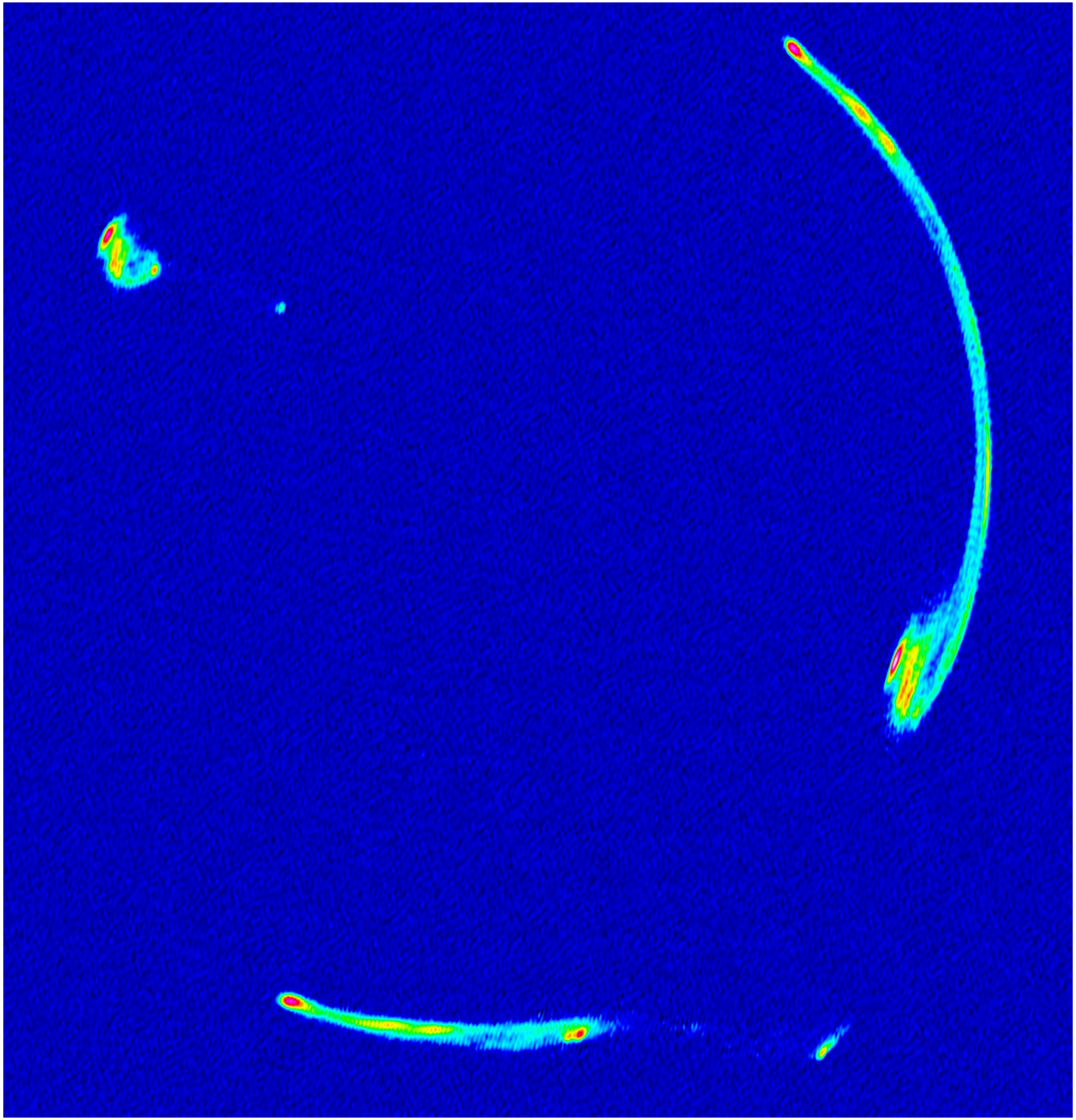
1.4 Astrophysical applications

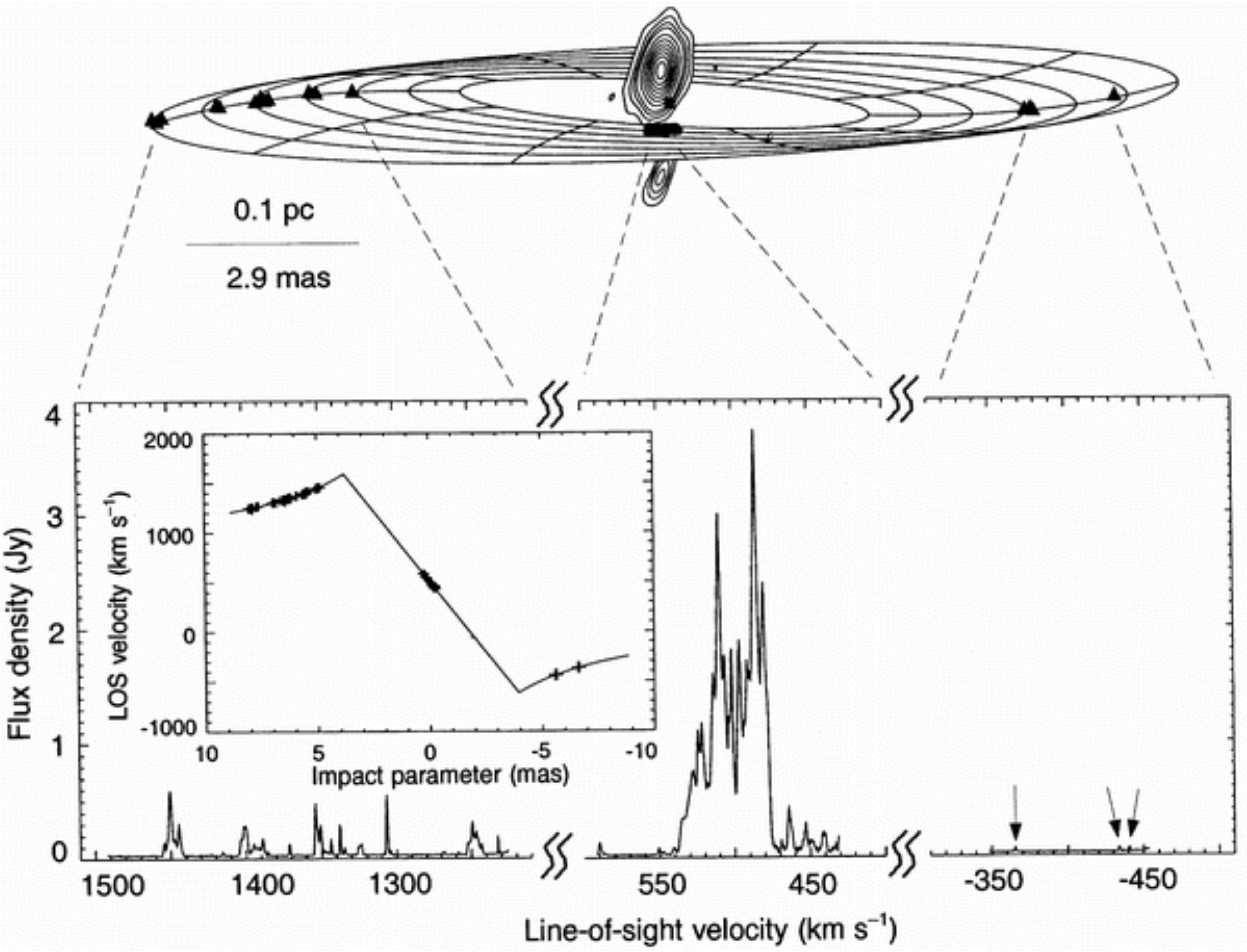
- The large radio window has allowed a wide variety of astronomical sources, thermal and non-thermal radiation mechanisms, and the propagation phenomena to be studied.

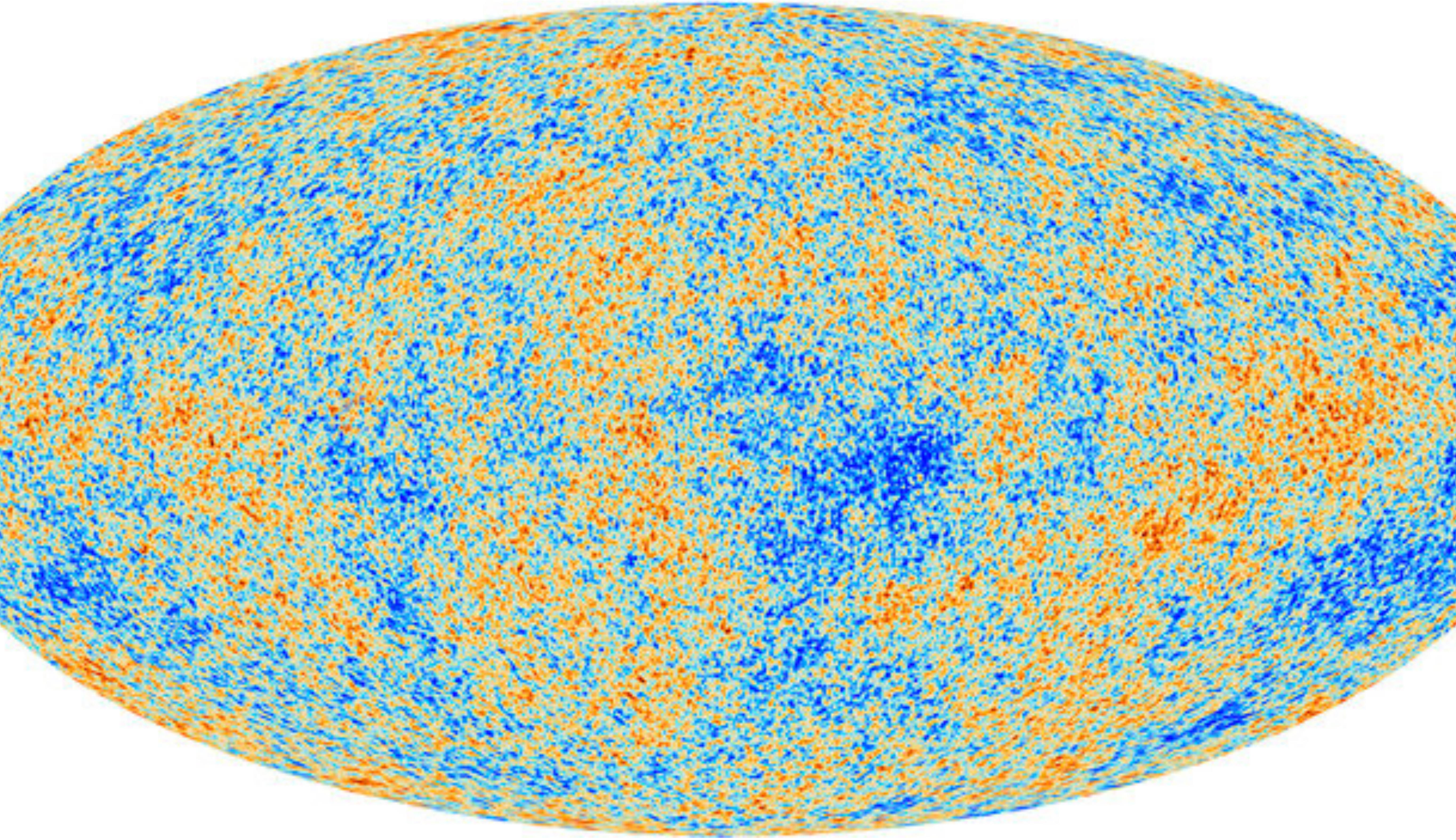
1. Discrete cosmic radio sources, at first, supernova remnants and radio galaxies (1948);
2. The 21cm line of atomic hydrogen (1951);
3. Quasi Stellar Objects "Quasars" (1963);
4. The Cosmic Microwave Background (1965);
5. Inter stellar molecules and pronto-planetary discs (1968);
6. Pulsars (1968);
7. Gravitational lenses (1979);
8. The Sunyaev-Zeldovich effect (1983);
9. Distance determinations using source proper motions determined from Very Long Baseline Interferometry (1993); and
10. Molecules in high-redshift galaxies (2005).



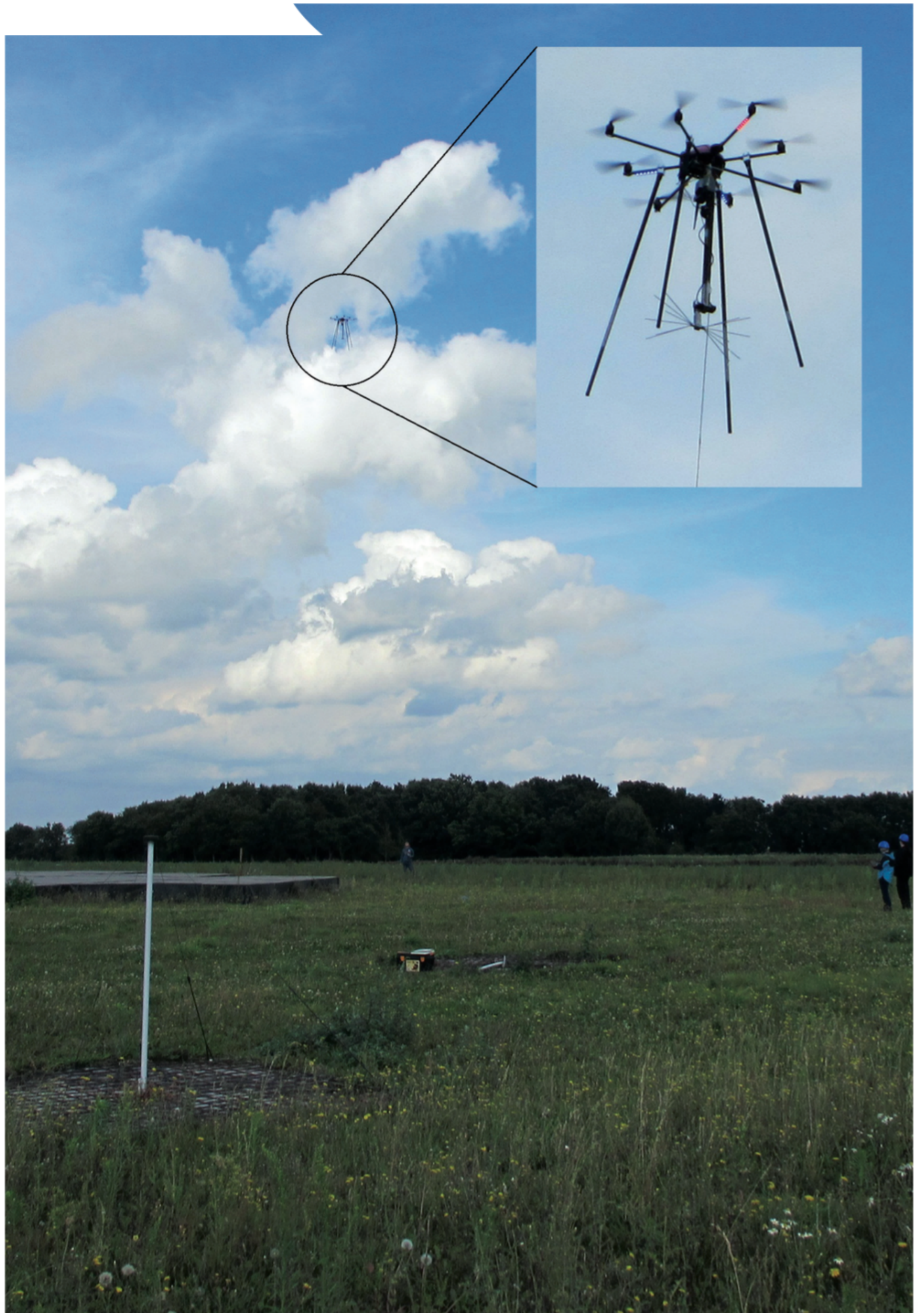


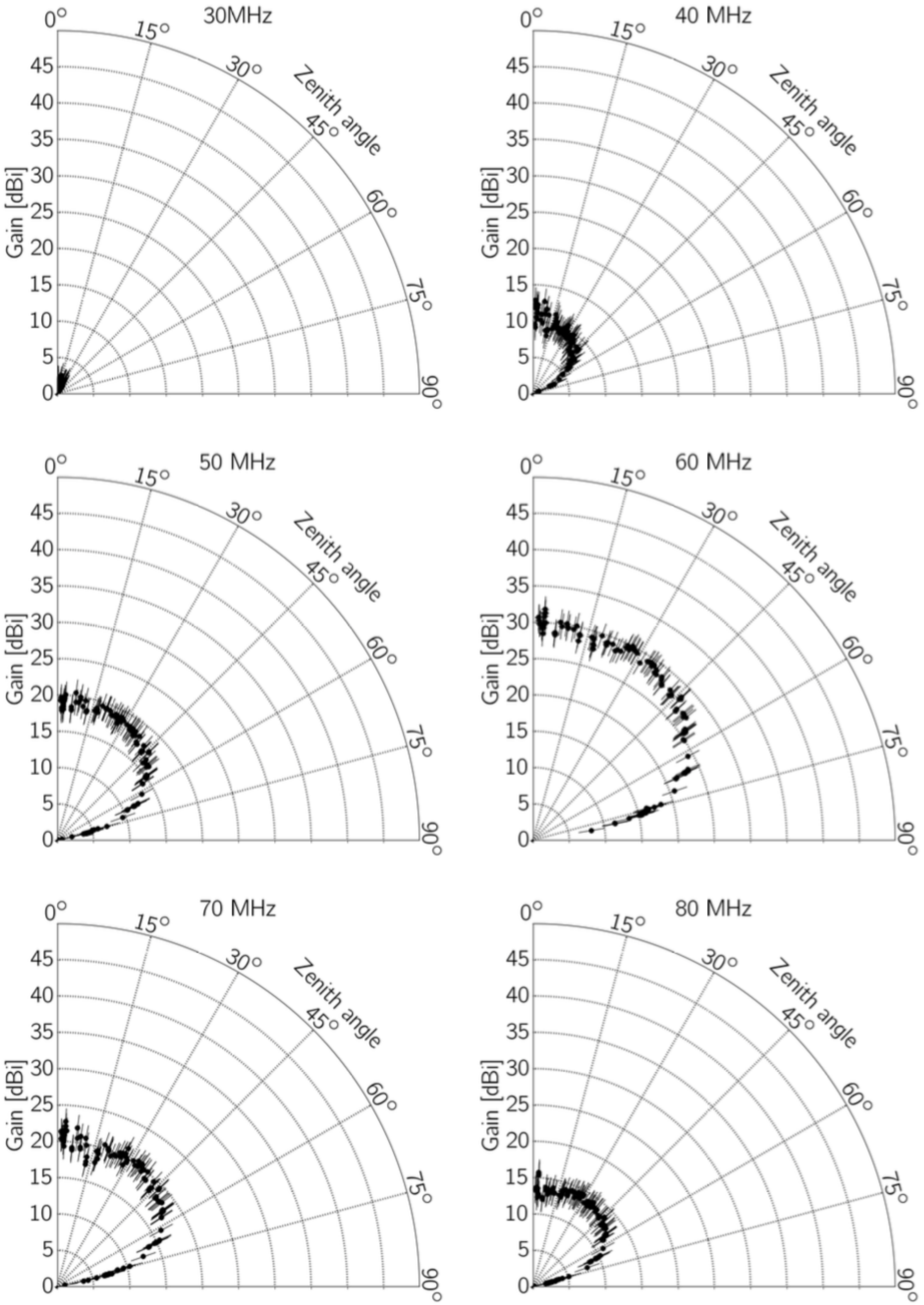



0.1 pc
2.9 mas
Flux density (Jy)
0
1
2
3
4
1500
1400
1300
550
450
-350
-450
Line-of-sight velocity (km s^{-1})
LOS velocity (km s^{-1})
2000
1000
0
-1000
10
5
0
-5
-10
Impact parameter (mas)



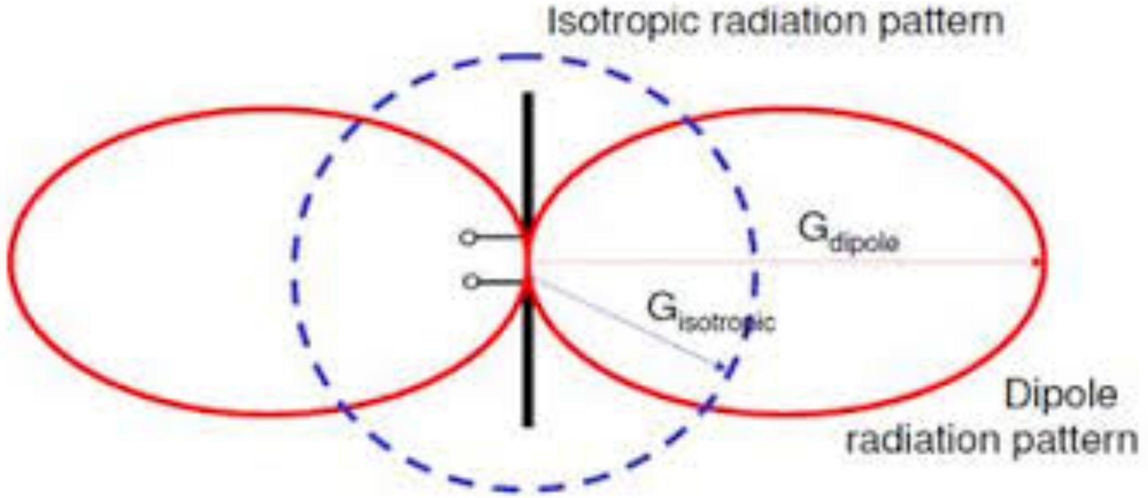
2.1 Response of the LOFAR antenna:





2.2 Power gain:

G(θ, ϕ) is the power transmitted per unit solid angle in direction *(θ, ϕ)* divided by the power transmitted per unit solid angle from an isotropic antenna with the same total power.



- The power or gain are often expressed in logarithmic units of decibels (dB):

$$G(\mathrm{dB}) \equiv 10 \times \log_{10}(\mathrm{G})$$

Worked example: What is the maximum and half power of a normalised power pattern in decibels?

Maximum power of a normalised power pattern is $P_n = 1$

$$P_n(1) = 10 \times \log_{10}(1) = 0 \text{ dB}$$

Half power of a normalised power pattern is $P_n = 0.5$

$$P_n(0.5) = 10 \times \log_{10}(0.5) = -3 \text{ dB}$$

For a lossless isotropic antenna, conservation of energy requires the directive gain averaged over all directions be,

$$\langle G \rangle \equiv \frac{\int_{\text{sphere}} G d\Omega}{\int_{\text{sphere}} d\Omega} = 1$$

Therefore, for an isotropic lossless antenna,

$$\int_{\text{sphere}} G d\Omega = \int_{\text{sphere}} d\Omega = 4\pi \quad \text{and} \quad G = 1$$

- Lossless antennas may radiate with different directional patterns, but they cannot alter the total amount of power radiated —> the gain of a lossless antenna depends only on the angular distribution of radiation from that antenna.

Key Concept: Higher the gain, the narrower the radiation pattern (directivity).

$$\Delta\Omega \approx \frac{4\pi}{G_{\max}}$$

- Beam solid angle: The beam area Ω_A is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its maximum value over Ω_A and zero everywhere else.

Normalised power pattern

Beam solid angle (sr)

sky area ($r^2 \sin \theta \, d\theta d\phi$)

$$\boxed{\Omega_{\mathrm{A}} \equiv \int_{4\pi} P_{\mathrm{n}}(\theta, \phi) d\Omega}$$

The power (and temperature) received is also a function of the power pattern of the antenna. Therefore, the true antenna temperature is,

$$\boxed{T_{\mathrm{A}} = \frac{A_{\mathrm{e}}}{2k} \int\int I_\nu(\theta, \phi) P_{\mathrm{n}}(\theta, \phi) d\Omega}$$

where $P_n(\theta, \phi)$ is the power pattern normalised to unity maximum,

$$P_{\mathrm{n}} = \frac{G(\theta, \phi)}{G_{\mathrm{max}}}$$

2.3 Response of a reflector antenna

- Paraboloidal reflectors are useful because,
  1. The effective collecting area A_e can approach the geometric area ($= \pi D^2/4$).
  2. Simpler than an array of dipoles.
  3. Can change the feed antenna to work over a wide frequency range (e.g. for the JVLA 8 receivers on each telescope allow observations from 1–50 GHz).

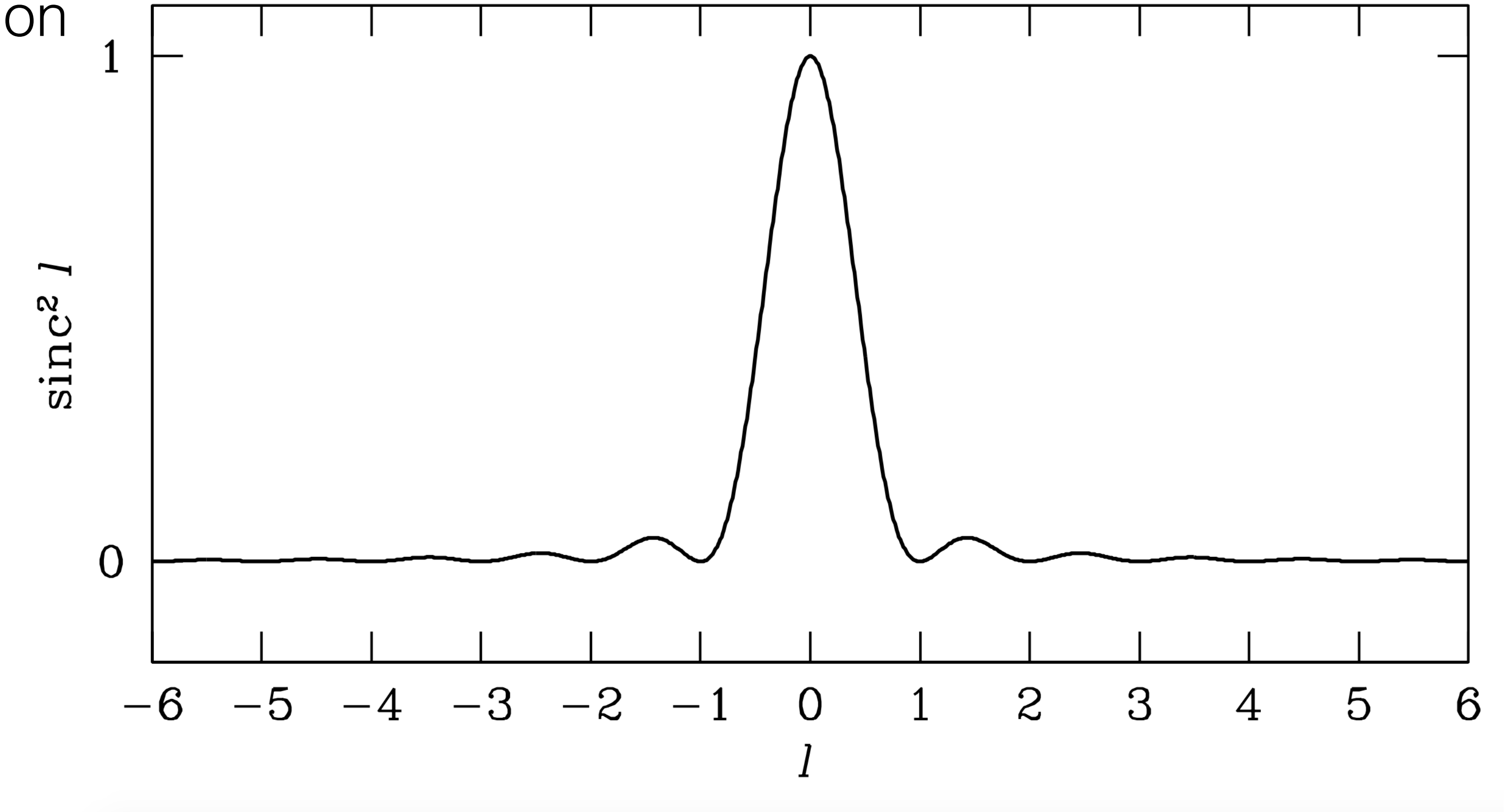
- For a receiving antenna, where the electric field pattern is *f(l)* and the electric field illuminating the aperture is *g(u)*,

$$\boxed{f(l) = \int_{\text{aperture}} g(u) e^{-i2\pi l u} du}$$

Key concept: In the far-field, the electric field pattern is the Fourier transform of the electric field illuminating the aperture.

- The radiated power as a function of position

$$\boxed{P_n(l) = \mathrm{sinc}^2\left(\frac{\theta D}{\lambda}\right)}$$

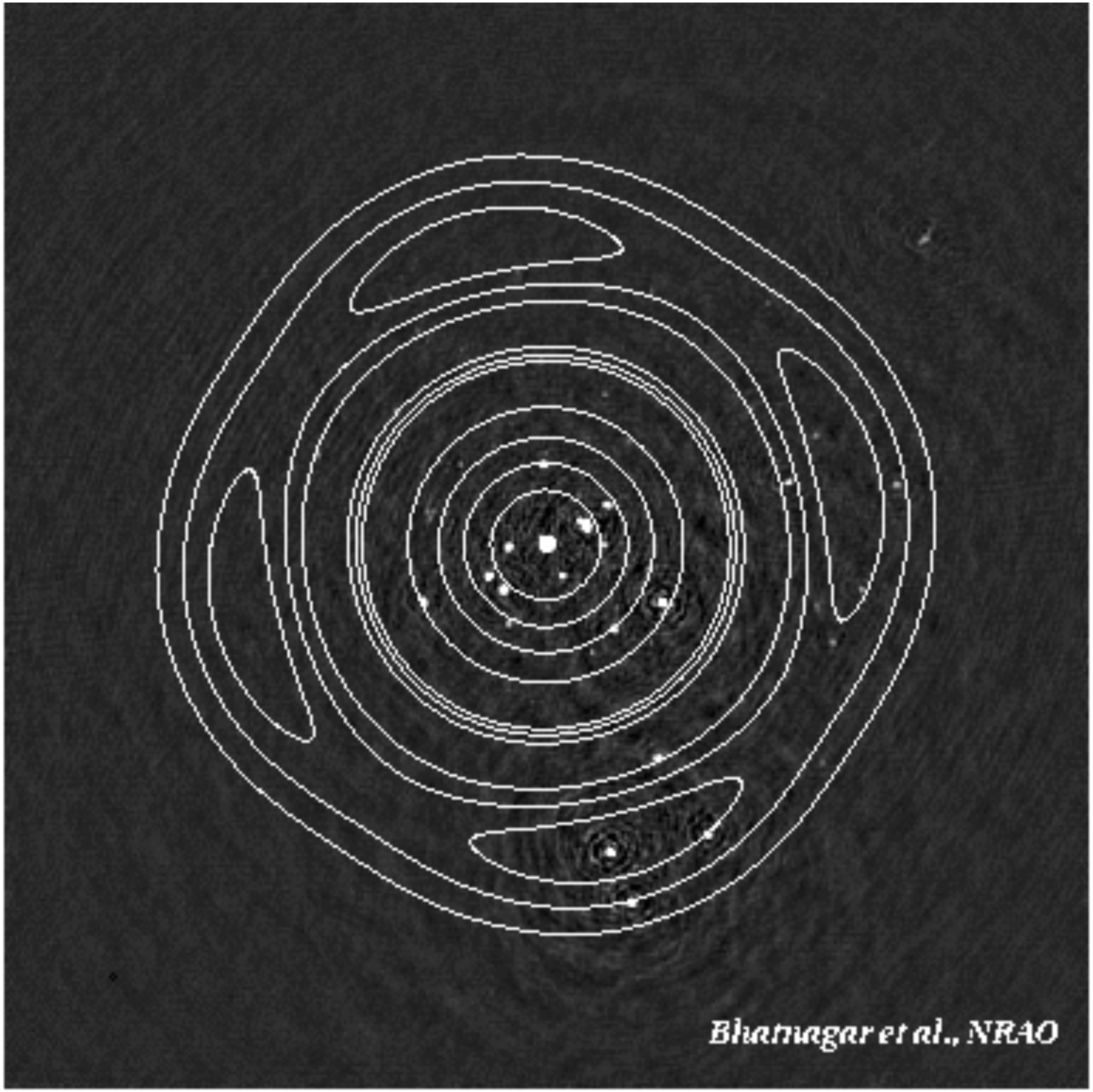


- For a one-dimensional uniformly illuminated aperture,

$$\boxed{\theta_{\mathrm{HPBW}} \approx 0.89\frac{\lambda}{D}}$$

- The central peak of the power pattern between the first minima is called the main beam (typically defined by the half-power angular size).

- The smaller secondary peaks are called sidelobes.



Bhatnagar et al., NRAO

- Main beam solid angle: The area containing the principle response out to the first zero.

- Side-lobes: Areas outside the principle response that are non-zero.

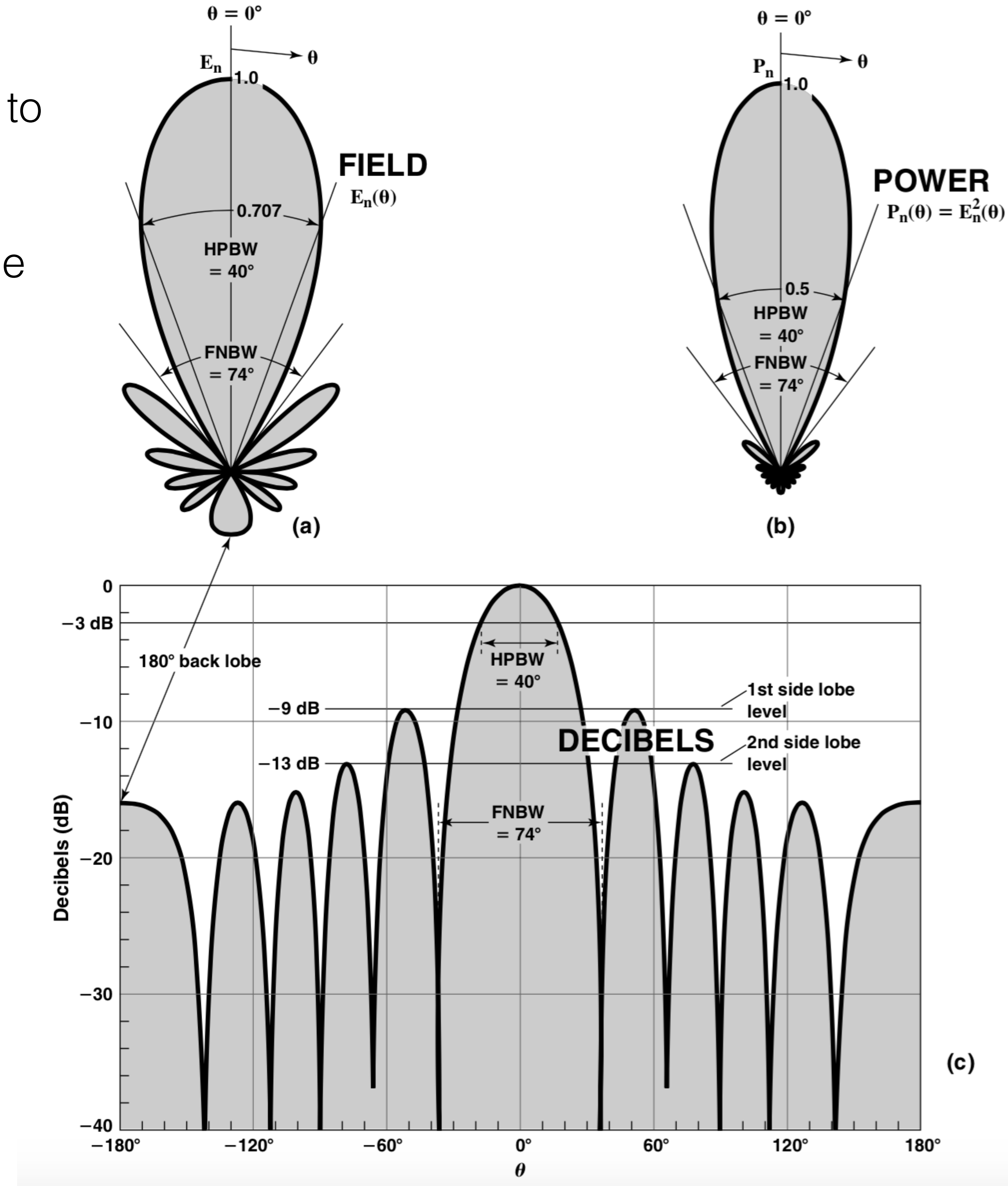
$$\boxed{\Omega_{\rm MB} \equiv \int_{\rm MB} P_{\rm n}(\theta,\phi)d\Omega}$$

Main beam solid angle (sr)

- Main beam efficiency: The fraction of the total beam solid angle inside the main beam

Main beam efficiency

$$\boxed{\eta_{\rm B} \equiv \frac{\Omega_{\rm MB}}{\Omega_{\rm A}}}$$



3.1 Sensitivity

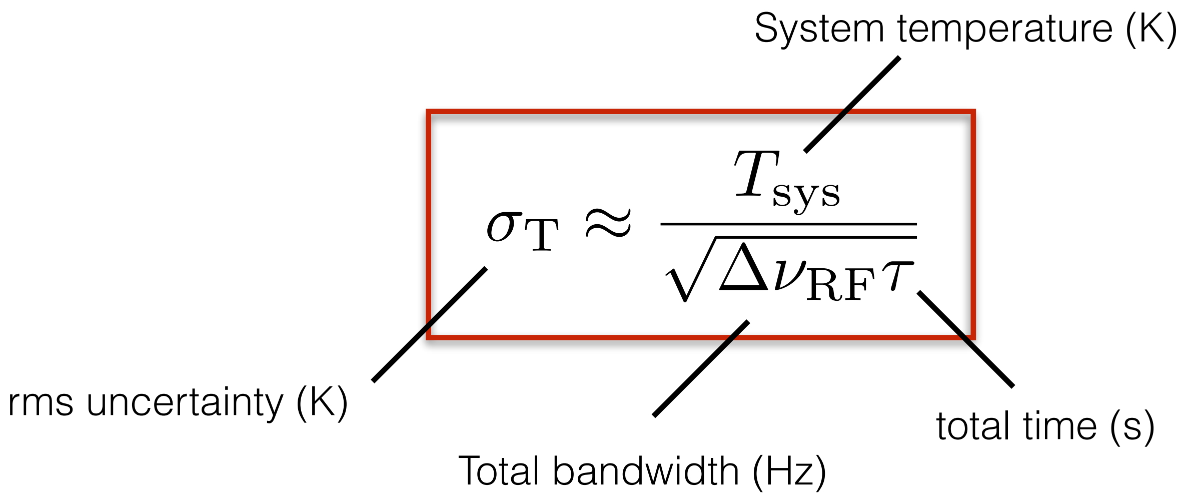
- Our ability to measure a signal is dependent on the noise properties of our complete system ($T_{\rm sys}$), although this is typically dominated by the Johnson noise within the receiver.

The variations (uncertainty on some measurement) is estimated by,

1. In time interval τ there are a minimum $N = 2\,\Delta\nu\,\tau$ independent samples of the total noise power $T_{\rm sys}$.
2. The uncertainty in the noise power (from a random gaussian distribution) is $\approx 2^{1/2}\,T_{\rm sys}$.
3. The rms error in the average of $N \gg 1$ independent samples is reduced by the factor $N^{1/2}$,

$$\sigma_{\rm T} = \frac{2^{1/2} T_{\rm sys}}{N^{1/2}}$$

which gives the (ideal) radiometer equation,



System temperature (K)

$$\boxed{\sigma_{\rm T} \approx \frac{T_{\rm sys}}{\sqrt{\Delta\nu_{\rm RF}\tau}}}$$

rms uncertainty (K)

total time (s)

Total bandwidth (Hz)

Typically $T_{\text{sys}} \gg T_{\text{source}}$. Need rms uncertainty in the system temperature to be as low as possible. Increase the observed bandwidth or observing for longer, or decrease the receiver temperature.

- The signal-to-noise ratio of our target source is,

$$\frac{S}{N} = \frac{T_{\text{source}}}{\sigma_{\text{T}}} = \frac{T_{\text{source}}}{T_{\text{sys}}} \sqrt{\Delta_{\text{RF}} \tau}$$

It is convenient to express the rms uncertainty in terms of the system equivalent flux density (SEFD; units of Jy). Recall

$$P_\nu = kT_{\text{A}} = A_e \frac{S_\nu}{2}$$

$$T_{\text{A}} = \left(\frac{A_e}{2k}\right) S_\nu$$

Called the 'forward gain' (K / Jy)

Define the SEFD as,

$$\boxed{\text{SEFD} = \frac{2kT_{\text{sys}}}{A_e}}$$

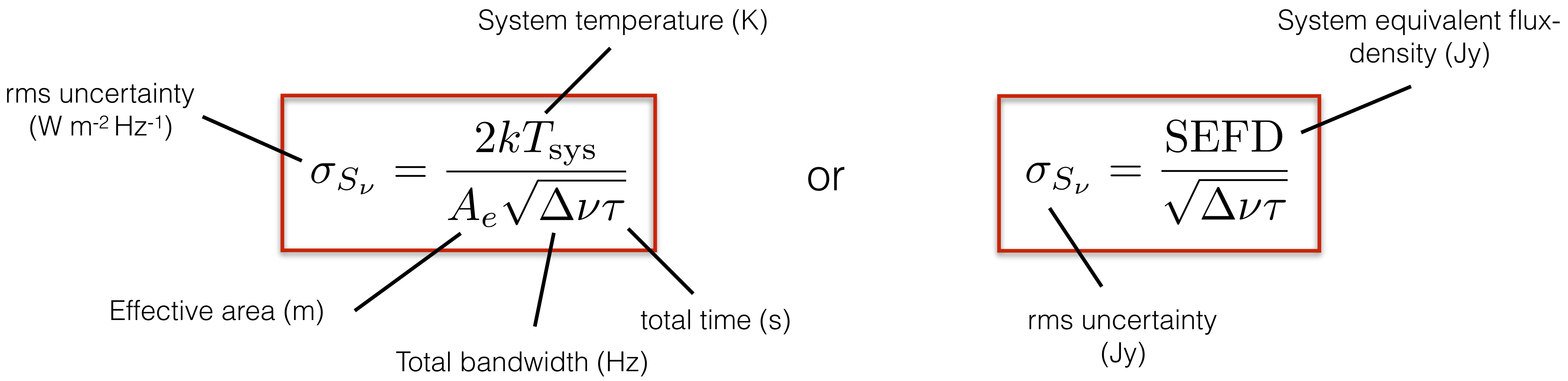
Worked example: What is the SEFD of a 25-m VLA antenna assuming a system temperature of 55 K and an effective area of 365 m^2?

$$\mathrm{SEFD} = \frac{2kT_{\mathrm{sys}}}{A_e} = \frac{2 * 1 \times 10^{-23} * 55}{365} = 3 \times 10^{-24}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$$
$$= 300\ \mathrm{Jy}$$

This will scale inversely with the effective area, i.e. a low SEFD suggests a more sensitive telescope.

- The SEFD is a good way to compare the sensitivity of telescopes because it takes the receiver system (T_{sys}) and the effective area (A_e) into account.

- We can define our ideal radiometer equation to determine the sensitivity in terms of flux-density,



System temperature (K)

rms uncertainty (W m^{-2} Hz^{-1})

$$\sigma_{S_\nu} = \frac{2kT_{\mathrm{sys}}}{A_e\sqrt{\Delta\nu\tau}}$$

Effective area (m)

Total bandwidth (Hz)

total time (s)

or

System equivalent flux-density (Jy)

$$\sigma_{S_\nu} = \frac{\mathrm{SEFD}}{\sqrt{\Delta\nu\tau}}$$

rms uncertainty (Jy)

Summary

1. Radio astronomy covers 5 decades in frequency from ~10 MHz up to about 1 THz (ground based).

2. It is a well established area of astronomical research that allows for a large number of unique science cases to be investigated (sensitivity and resolution).

3. Keep in mind the properties of the single elements of your interferometer.

Enjoy your week at ERIS 2017

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